Learning to be Risk Averse?

Robert Marks
Economics, University of New South Wales,
and University of Melbourne

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robert.marks@gmail.com

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Roadmap.

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2. Decisions Under Uncertainty and Risk Profile
3. Utility Functions
    a. Constant Absolute Risk Aversion
    b. Constant Relative Risk Aversion
4. The Simulations and Results
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1. Introduction

What is the best risk profile for a decision-maker in a risky world to have?

In a von Neumann-Morgernstern world there are three (constant) possibilities:

1. **risk-neutral: choose the prospect with the highest expected value**

2. **risk-averse: choose the prospect with the highest expected utility**

3. **risk-preferring: ditto**

By risky is meant that the possible outcomes and probabilities (which might be subjective) are known.

Informally, it is widely held that in a risky world, with the possibility of the discontinuity of bankruptcy, the most prudent risk profile is risk aversion.

"Risk aversion is one of the most basic assumptions underlying economic behavior" (Szpiro 1997), perhaps because "a dollar that helps us avoid poverty is more valuable than a dollar that helps us become very rich" (Rabin 2000).

But is risk aversion the best risk profile? Even with bankruptcy as a possibility?

Previous researchers' answers:

- **Szpiro (1997): risk averse**
- **Chen et al. (2005): risk averse (log utility)**
- **and DellaVigna & LiCalzi (2001) model Kahneman-Tversky agents which learn to make risk-neutral choices.**

Methodology

We use two kinds of von Neumann-Morgenstern utility function (the wealth-independent exponential utility function, or Constant Absolute Risk Aversion CARA, and the Constant Relative Risk Aversion CRRA function, which is sensitive to the agent's level of wealth) and run computer experiments in which each agent chooses among three lotteries, and is then awarded with the outcome of the chosen lottery k .

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Repetition of this choice by many agents allows us use a technique from machine learning — the Genetic Algorithm (Holland 1992) — to search for the best risk profile, where “best” means the highest average payoff when chosing among lotteries.

2. Decisions under Uncertainty and Risk Profiles

The von Neumann-Morgenstern formulation of the decision-maker's attitude to risk is based on the observation that individuals are not always expected-value decision makers. That is, there are situations in which people apparently prefer a lower certain outcome to the higher expected (or probility-weighted) outcome of an uncertain prospect (where the possible outcomes and their possibly subjective, or Bayesian, probabilities are known).

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An example is paying an insurance premium that is greater than the expected loss without insurance. On the other hand, people will sometimes "gamble" by apparently preferring a lower uncertain outcome to a higher sure thing: this is risk-preferring.

We can formalise this by observing that, by definition, the utility of a lottery is its expected utility, or

$$U(\mathrm{L}) = \sum p_i U(x_i), \tag{1}$$

where each (discrete) outcome x_i occurs with probability p_i, and $U(x_i)$ is the utility of outcome x_i.

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Approximating the Certainty Equivalent

Expand utility $U(.)$ about the expected value $\bar{x}$.

$$U(x_0) \approx U(\bar{x}) + (x_0 - \bar{x})U'(\bar{x}) + \tfrac{1}{2}(x_o - \bar{x})^2 U''(\bar{x})$$

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Therefore, from equations (4) and (5),

$$\begin{aligned} \tilde{x} - \bar{x} &\approx \tfrac{1}{2}\sigma^2 \frac{U''(\bar{x})}{U'(\bar{x})} \\ \therefore\ \tilde{x} &\approx \bar{x} + \tfrac{1}{2}\sigma^2 \frac{U''(\bar{x})}{U'(\bar{x})} \end{aligned} \qquad (6)$$

3. Utility Functions

We consider two types of von Neumann-Morgenstern utility function:

1. **those which exhibit constant risk preference across all outcomes (so-called wealth-independent utility functions, or Constant Absolute Risk Aversion CARA functions), and**
2. **those where the risk preference is a function of the wealth of the decision maker (the Constant Relative Risk Aversion CRRA functions).**

   This includes the risk-averse logarithmic utility function.

Wealth Independence

If an increase of all outcomes in a lottery by an equal amount Δ increases the C.E. of the lottery by Δ, then the decision maker exhibits wealth independence:

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CARA utility functions charactise risk preference by a single number, the *risk aversion coefficient*, γ.

Since CARA utility functions are wealth-independent, any aversion to bankruptcy is thus precluded, by definition.

Whether a decision maker exhibits a wealth-independent utility function is an empirical question.

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$$U(x) = 1 - e^{-\gamma x}, \tag{7}$$

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where $U(0) = 0$ and $U(\infty) = 1$, and where γ is the *risk aversion coefficient*:

$$\gamma \equiv -\frac{U''(x)}{U'(x)}. \tag{8}$$

Risk Aversion with Exponential Utility

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From equations (6) and (8), for exponential utility,

$$\tilde{x} \approx \bar{x} - \tfrac{1}{2}\sigma^2\gamma$$

which indicates that when $\gamma = 0$, then $\tilde{x} \approx \bar{x}$ (risk neutrality), when $\gamma > 0$, then $\tilde{x} < \bar{x}$ (risk averse), and when $\gamma < 0$, then $\tilde{x} > \bar{x}$ (risk preferring), with positive variance.

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Summarizing this:

Sign of γ	Risk profile	Curvature
$\gamma = 0$	risk neutral	$U''(x) = 0$
$\gamma > 0$	risk averse	$U''(x) < 0$
$\gamma < 0$	risk preferring	$U''(x) > 0$

3.2 CRRA Utility Functions

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The Arrow-Pratt measure of relative risk aversion (RRA) ρ is defined as

$$\rho(w) = -w\,\frac{U''(w)}{U'(w)} = w\gamma \qquad (9)$$

This introduces wealth w into the agent's risk preferences, so that lower wealth can be associated with higher risk aversion. Risk aversion coefficient γ is as in equation (8).

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The Constant Elasticity of Substitution (CES) utility function:

$$U(w) = \frac{w^{1-\rho}}{1-\rho}, \quad w > 0, \qquad (10)$$

exhibits constant relative risk aversion CRRA, equation (9).

When $\rho \rightarrow 1$, (10) becomes logarithmic: $u(w) = \ln(w)$.

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With $w > 0$, $\rho > 0$ is equivalent to risk aversion.
With $w > 0$ and $\rho = 1$, the CES function becomes the (risk-averse) logarithmic utility function, $U(w) = \log(w)$.
With $w > 0$ and $\rho < 0$ is equivalent to risk preferring.

4. The Simulations

Each lottery is randomly constructed: the two payoffs ("prizes") are randomly chosen in the interval between − and + Maximum Absolute Prize (MAP), usually 100; and the probability is also chosen randomly.

Each agent calculates the expected utility of each of the three lotteries, using its utility function (a function of its γ or ρ/W), and chooses the lottery k with the highest expected utility. To do this, agents know the prizes and probabilities of all three lotteries.

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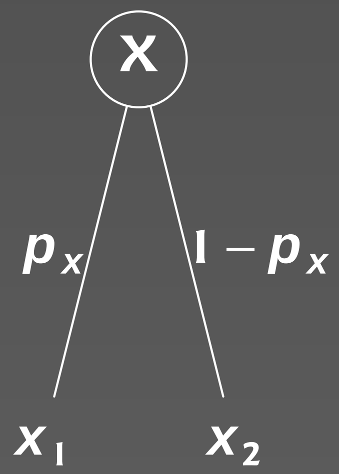
Diagrammatically

Three two-prize lotteries with random prizes and probability:



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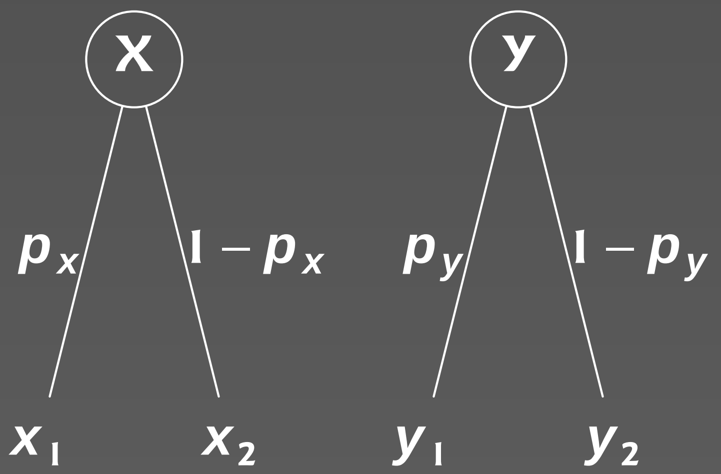

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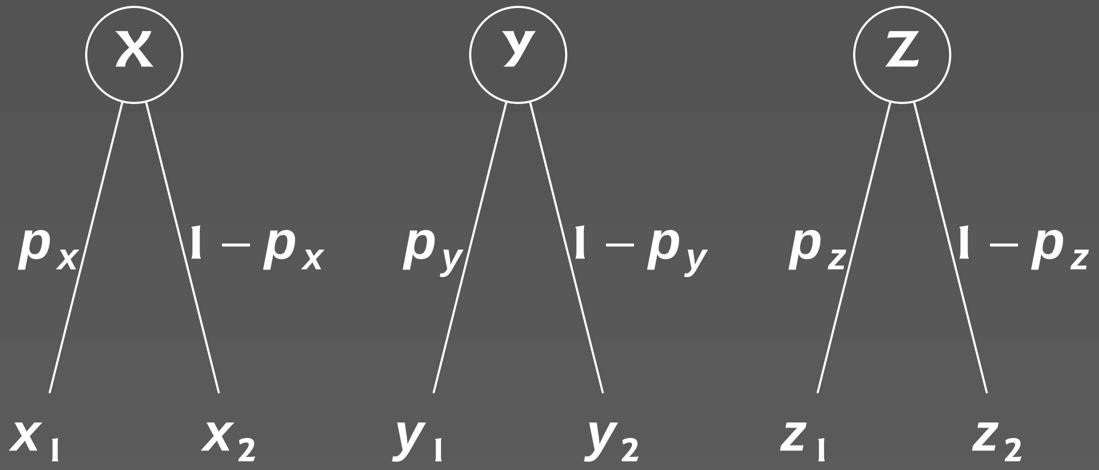

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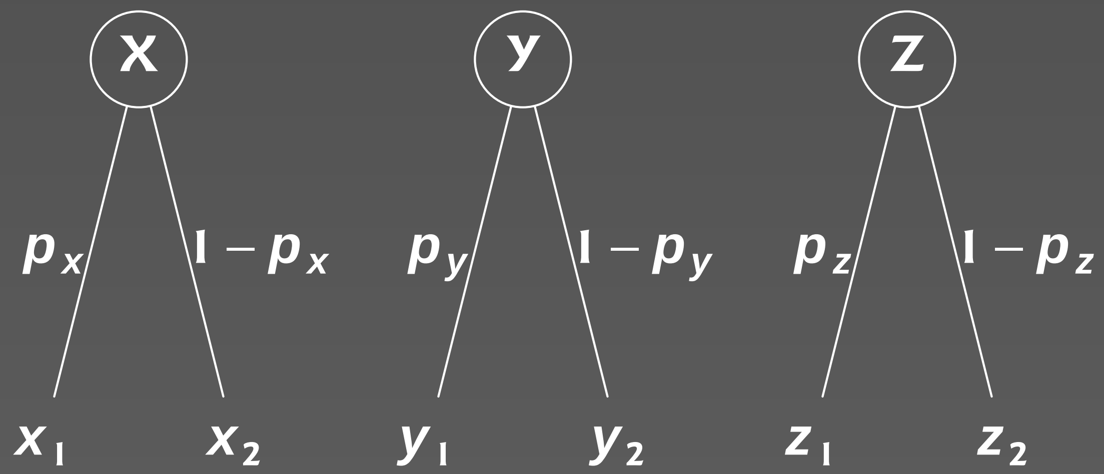

Calculate the three expected utilities, functions of γ (or ρ and w):

$$U(X) = p_x U(x_1) + (1 - p_x) U(x_2)$$
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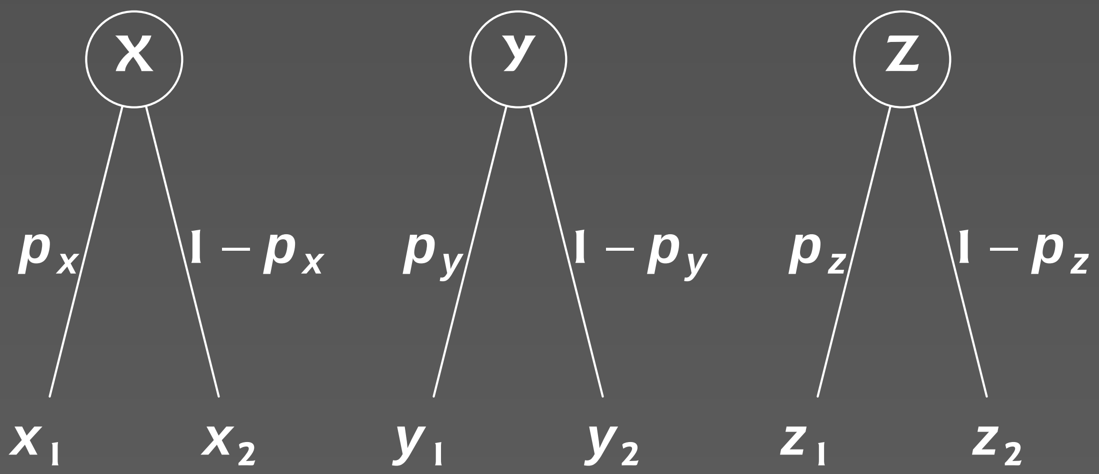

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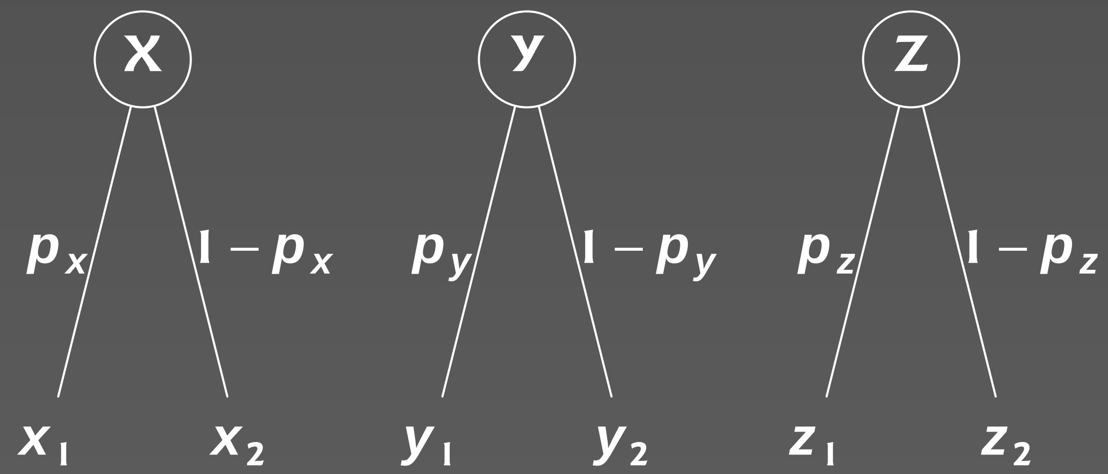

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Choose the lottery I with the highest expected utility. Win whichever prize (i_1 or i_2) is realised in that lottery, based in the lottery's probability p_i.

Searching with the Genetic Algorithm

We now use an implementation of the Genetic Algorithm (Gilbert 2004) to search for the best risk profile. That is, we select the best-performing agents to be the "parents" of the next generation of agents, which is generated by "crossover" and "mutation" of the chromosomes of the pairs of parents. Each of the new generation of agents chooses the lottery k with highest expected utility a thousand times. Again, the best are selected to be the parents of the next generation.

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We use the GA simulation in this search as an empirical alternative to solving for the best (highest performing) risk profile analytically. Note that Rabin (2000) asserts that "theory actually predicts virtual risk neutrality." We return to this paper in the Discussion below.

4.1 The Simulations with CARA Utility

Using NetLogo (Wilensky 1999), we model each agent as a binary string which codes to its risk-aversion coefficient, γ, in the interval $\pm$ 1.048576.

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Each agent chooses the lottery k with the highest expected utility from equations (1) and (7), based on its value of γ. Then a realised outcome is calculated for that lottery, based on its probability.

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Each agent faces 1000 lottery choices, and the cumulative winnings that agent's "fitness" for the Genetic Algorithm.

The CARA Results

See `http://www.agsm.edu.au/bobm/teaching/SimSS/NetLogo4-models/RA-CARA-EU-3l2p.html` **for a Java aplet and the NetLogo code.**

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The windows captured from the NetLogo simulations show three things clearly:

1.

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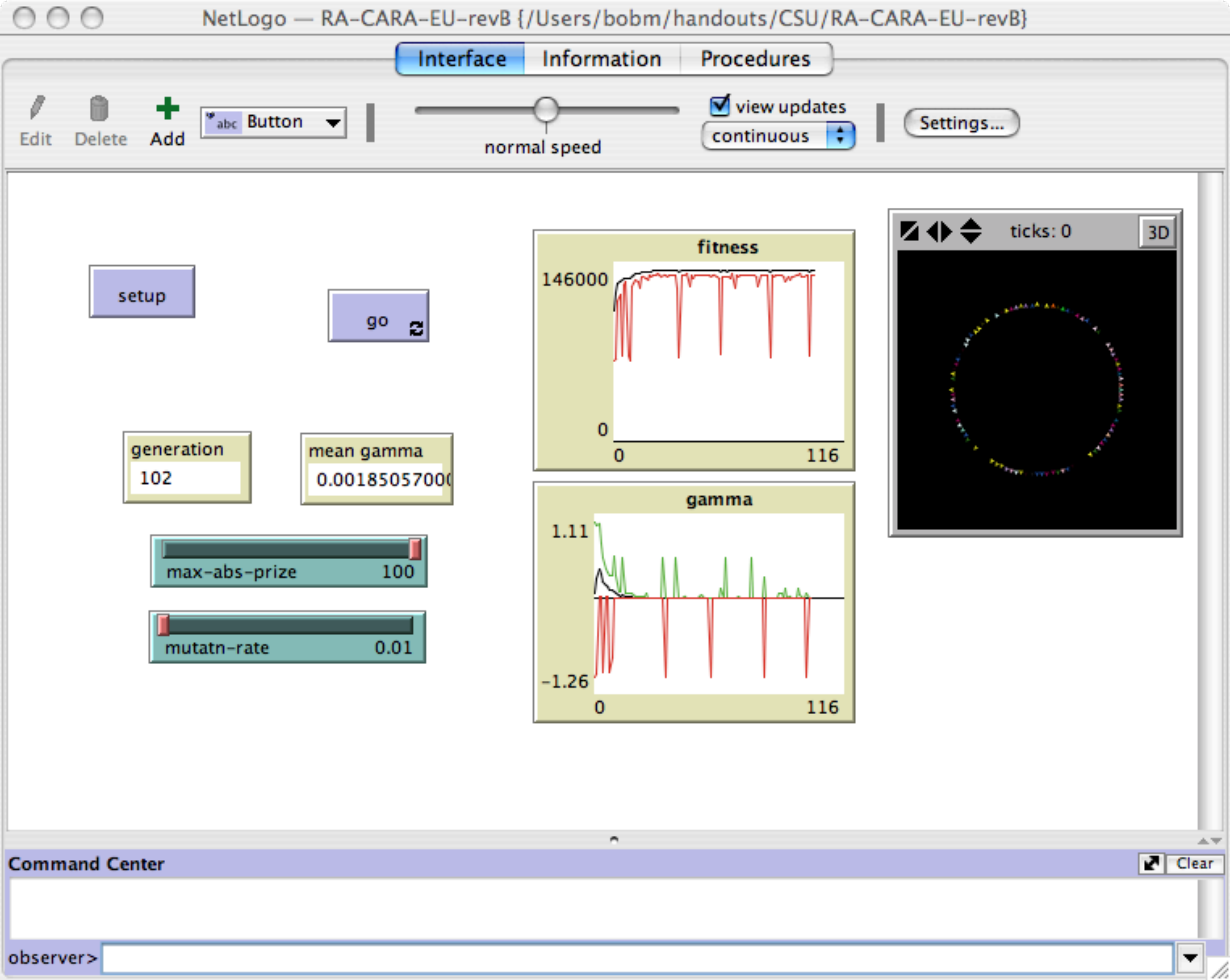
Too risk averse, and they forgo fair lotteries; too risk preferring and they choose too many risky lotteries.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4.2 The CRRA Results

See `http://www.agsm.edu.au/bobm/teaching/SimSS/NetLogo4-models/DRA-CRRA-EU-revCD-3l2p.html` **for a Java aplet and the NetLogo code.**

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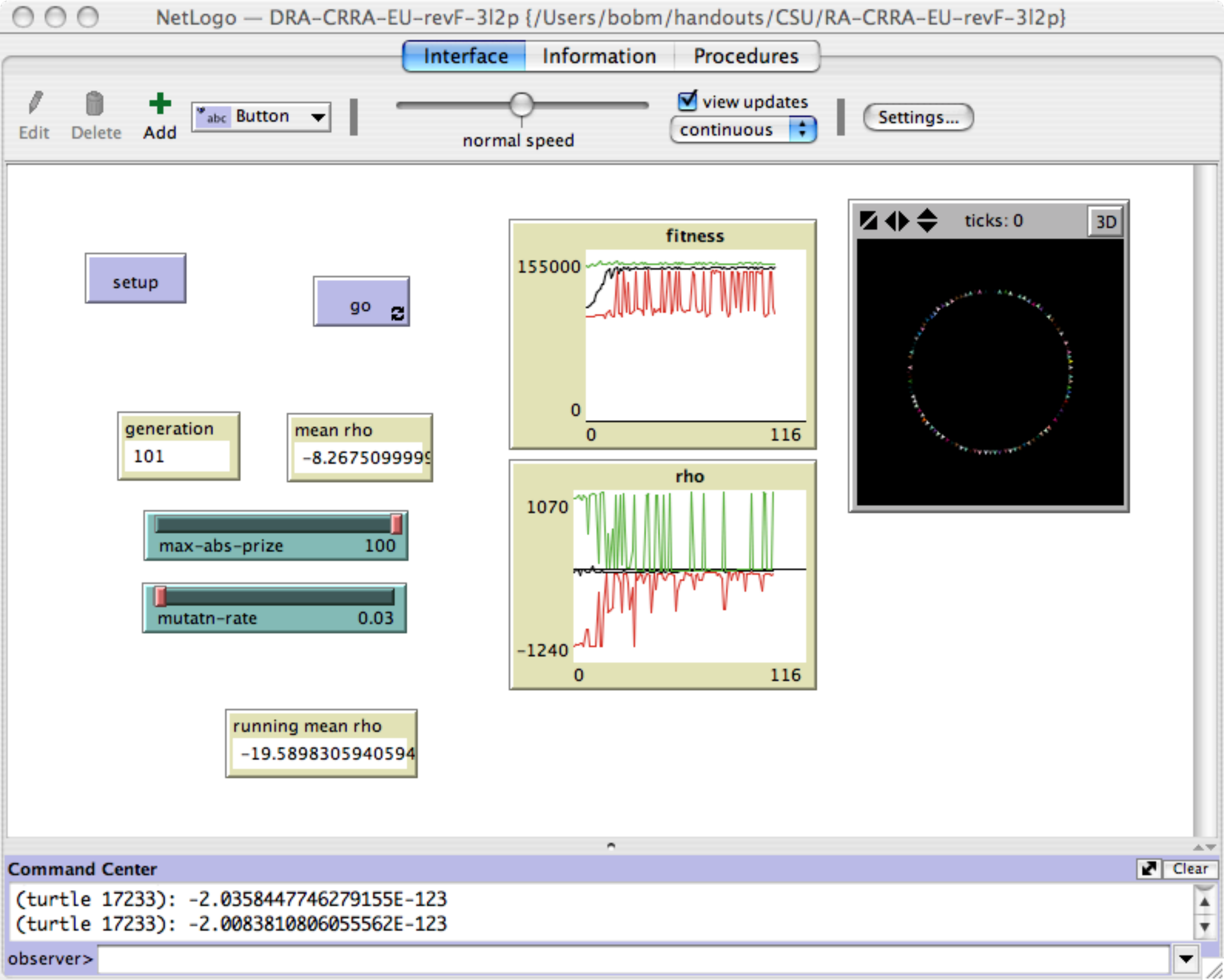
The results are surprising: the CRRA agents do not learn to be risk averse, but are very close to risk neutral.

Remember: $\gamma = \frac{\rho}{W}$, so dividing the ρ values by the high W values attained implies corresponding minute values of γ here.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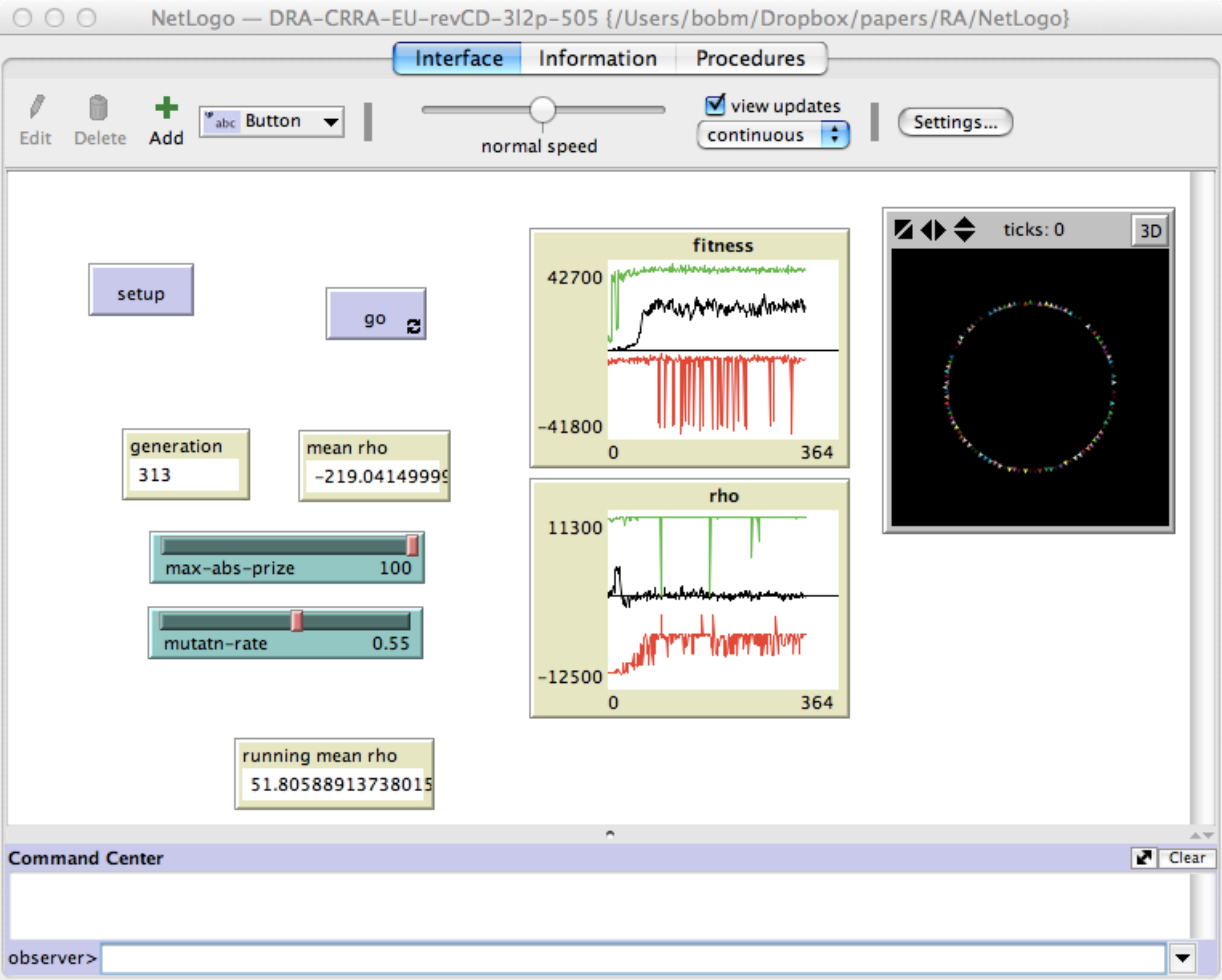
Choice under Uncertainty

So far: decision-making under risk (knowing payoffs and probabilities). What of decision making under uncertainty?

Let's blind the agents to the probabilities. They choose a lottery making the (wrong) Laplacian assumption of equi-probable prizes.

With CARA and CRRA utility, the mean γ is close to zero (risk neutral) but the outliers don't converge.

This might explain Chen & Huang (2005)'s results of log-utility decision makers (i.e. risk averse) performing best in an artificial stock market.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observer>

5. Discussion

Unlike the GA simulations of Szpiro (1997), we find that the best-performing CARA agents are risk-neutral, not risk averse. Because of the indirect way in which Szpiro modelled the risk profiles of his agents (unlike a referee's suggestion, footnote 3, Szpiro's model "only distinguishes between risk-averse automata and all others"), explanation of the contradictory results is not easy, but since our models allow any risk profile to emerge, we argue that they are more general than Szpiro's.

5. Discussion

Unlike the GA simulations of Szpiro (1997), we find that the best-performing CARA agents are risk-neutral, not risk averse. Because of the indirect way in which Szpiro modelled the risk profiles of his agents (unlike a referee's suggestion, footnote 3, Szpiro's model "only distinguishes between risk-averse automata and all others"), explanation of the contradictory results is not easy, but since our models allow any risk profile to emerge, we argue that they are more general than Szpiro's.

Should we be surprised that risk neutrality does better than risk aversion in CARA utility functions? Rabin (2000) suggests a reason why not, at least for small-stakes lotteries. He argues that von Neumann-Morgenstern expected-utility theory is inappropriate for reconciling actual human behaviour as revealed in risk attitudes over large stakes and small stakes. If there is risk aversion for small stakes, then expected-utility theory predicts wildly unrealistic risk aversion when the decision maker is faced with large stakes. Or risk aversion for large stakes must be accompanied by virtual risk neutrality for small stakes.

Rabin argues that *loss aversion* (Kahneman and Tversky 1979), rather than risk aversion, is a better (i.e. more realistic) explanation of how people actually behave when faced with risky decisions. This suggests possibilities for further simulations, although "loss aversion" suggests a prior conclusion.

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But we do not appeal to empirical evidence or even to prior beliefs of what sort of risk profile is best. Whereas there has been much research into reconciling actual human decision making with theory (see Arthur 1991), we are interested in seeing what is the best (i.e. most profitable) risk profile for agents faced with risky choices.

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And we find that for wealth-independent CARA utility functions (exponential) agents learn to become risk-neutral decision makers in order to maximise their returns when choosing among risky propositions. This is different from the risk-averse agents that Szpiro

(1997) observed. But for wealth-dependent CRRA utility functions (CES) our agents often do learn to be slightly risk averse, as expected, but not always.

Acknowlegements

I should like to thank Simon Grant, Luis Izquierdo, the participants of the Complex Systems Research Summer School 2007 at Charles Sturt University, the participants at the 26th Australiasian Economic Theory Workshop 2008 at Bond University, Jasmina Arifovic, James Andreoni, Seth Tisue, and Nigel Gilbert for his implementation of the Genetic Algorithm in NetLogo.

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