# Do stylised facts of order book markets need strategic behaviour?

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**Abstract**

This paper studies the role of strategy and the order book market mechanism in price dynamics and the order flow behaviour. To this end we analyse a zero-intelligence agent model of a dynamic limit order market. Stylised facts of limit order markets are shown to be influenced and, in some cases, governed by the market mechanism rather than strategic interaction. Positive correlation in order types, for instance, is the result of the market architecture, and price movements may be predicted in the short term from analysing the state of the order book. In contrast the absolute probabilities of order submission highlight the contribution of strategic behaviour.

# 1 Introduction

The strategic behaviour of traders is generally thought to be the dominant force influencing market dynamics, however, many of the key properties of double-auction markets such as high allocative efficiency are solely due to the constraints imposed by the trading mechanism, or market architecture, as forcefully demonstrated in Gode and Sunder (1993, 1997)'s seminal contributions. Their 'zero-intelligence' approach is as simple as it is convincing: in a market in which traders' behaviour is random, regularities must stem from the mechanism governing trade.

This paper furthers this line of enquiry by studying a dynamic limit-order driven market in which the entrance and exit of traders generates a continuous change in the population of agents. Empirical and analytical studies have both demonstrated and offered possible explanations for the presence of regularities in the order submission process and order book dynamics. By employing a model based on random individual behaviour we wish to determine if these regularities stem from traders' strategic behaviour or if they are a natural feature of order book markets. The submission of these orders and their possible subsequent execution give rise to the shape and dynamic behaviour of the order book. Characteristics of the order book shape (depths, spreads and tick size dependence) will be analysed to determine which match empirical evidence and which may be the result of trader strategy. From examining the order book dynamics we also wish to come to an understanding of the circumstances in which the order book provides predictive power. The order book is known to contain information about future price movements. This study aims to determine the source of this information. Is it as a result of the market mechanism or is it as a result of traders divulging their private information through their actions? A zero intelligence approach will allow us to answer these questions and provide us with insight into the driving forces of observed limit order behaviour, whether that be trader strategy or the market mechanism.

Issues regarding how limit orders are submitted by traders and how the limit order book affects trade have been studied in several papers. Empirical studies of limit order markets have documented their behaviour both in pure limit order markets (e.g. Biais et al. (1995) on the Paris Bourse, Griffiths et al. (2000) on the Toronto Stock Exchange and Hall and Hautsch (2006) on the Australian Stock Exchange), markets with specialists (e.g. Huang and Stoll (1994), Harris and Hasbrouck (1996) and Harris and Panchapagesan (2005)) and the contrast between the two (e.g. Lehman and Modest (1994)'s comparison of the NYSE and the Tokyo stock exchanges). The exact trading mechanisms of the above markets can differ markedly, however, some points seem common amongst many of the markets. For instance several authors (e.g. Hasbrouck, 1991a; Hamao and Hasbrouck, 1995; Biais et al., 1995) report positive correlation in order submission (i.e. orders of the same type following each other), that orders away from the best quotes have relatively little effect on market behaviour (e.g. Griffiths et al., 2000; Harris and Panchapagesan, 2005) and that the order book carries information about future market movements (e.g. Hall and Hautsch, 2006; Huang and Stoll, 1994). In our study we will principally be focusing on explaining the behaviour of non-specialist markets. The reason for this is that in pure order book markets only the interaction between the traders and the order book have to be

considered, the addition of the specialist, however, adds extra interactions that initially we would prefer to avoid considering.

Even in the more simple system of a pure order book market explaining these behaviours has not been straightforward. Experimental work has made contributions in this area: work by Campbell et al. (1991) has shed light on factors effecting the variability in the bid-ask spread and work by Smith et al. (1988) has provided possible explanations for the occurrences of bubbles and crashes in financial markets.

Analytical work has also attempted to explain some of this observed behaviour (e.g. Cohen et al., 1981; Foucault, 1999; Hollifield et al., 2004). For instance the work of Glosten (1994) provided a theory of the effect of order on price movements and as a consequence the relative profitability of orders. Parlour (1998) developed a model based on optimal order submission which provided the relative probabilities of buy and sell market and limit orders in different situations. Chakravarty and Holden (1995) develop an optimal strategy for order submission and show that in certain circumstances it can be optimal to submit limit orders on both sides of the market. There are, however, difficulties in forming these models, the order book by its very nature is relatively difficult to analyse. The sheer complexity of trader decisions and their effects has meant that analytical models frequently have to make significant simplifying assumptions in order to derive results. For instance restricting the price grid to very few prices (Parlour, 1998), only allowing a single market order (Handa and Schwartz, 1996), or potentially infinite liquidity (Seppi, 1997). For these reasons a computational approach is very attractive for studying this system.

Much of this analytical work has been based around the presence of rational traders, however, work involving irrational traders has also provided much insight. Taking inspiration from Becker (1962)'s work on budget constrained traders behaving randomly, Gode and Sunder (1993) were able to use computational simulation to explain much of the efficiency and convergence of the double auction market mechanism. This work has also been employed in settings closer to financial markets by Bollerslev and Domowitz (1992) who added an order book to the original work of Gode and Sunder. They found that the addition of a limit order book entailed an improvement in efficiency as well as in the convergence to the competitive equilibrium price. The first, however, experiences only a minor improvement because all the cases considered in Gode and Sunder (1993) exhibit a high degree of allocative efficiency. Bollerslev and Domowitz (1992) also obtain positive results in cases in which the individual demand and supply functions show less symmetry as in Gode and Sunder (1993). Cliff and Bruten (1997) give a fuller discussion of the effects of asymmetry in supply and demand within

3

the Gode and Sunder model.

Simulations developing the zero intelligence approach have been used to investigate the stylised facts present within financial markets (Bak et al., 1997; Maslov, 2000; Challet and Stinchcombe, 2001). Zero intelligence models have even been used analytically, though a quite radical departure from the simple and intuitive Gode/Sunder framework seems to be required. For instance the work of Farmer et al. (2005), which can reproduce the spread variance and price diffusion rate observed in the London Stock Exchange with just two parameters.

Our model will share features with the models of both Gode and Sunder (1993) and Bollerslev and Domowitz (1992) (though also see Farmer et al. (2005) and Duffy and Unver (2006) for related work), whilst developing the framework to provide better insight into financial markets through allowing orders with price and quantity as well as continuous trading through market entry and exit of traders.

In both of the models mentioned above traders are represented as a finite set of individuals each with it's own supply or demand function comprising the finite set of units that trader is able to trade. Once all of the intra-marginal units are traded the market converges and all trading stops. This behaviour is not observed in real financial markets, there is a constant flux of traders as new traders enter the market and old traders either complete their desired transactions or leave having not been able to complete them. In order to consider extended time frames it is necessary to resolve this issue.

Some zero intelligence models employed in finance research (e.g. Farmer et al., 2005; Maslov, 2000) have avoided the problem of cessation of trade by substituting individual traders with an order submission process which provides a constant stream of new limit orders. This solution, however, does not allow for the effect of trade on the supply and demand functions of individual traders and consequently one of the important market structural effects is missing. In this model we will employ a new solution, the arrival and departure of traders from the market will be explicitly modeled. This process will operate with the minimum number of assumptions in order to maintain the models simplicity and to avoid creating artifacts in the results. In essence we will introduce new traders at random and similarly remove existing traders.

One of the main strengths of order book financial markets is that they allow traders to submit offers for the quantities of assets they desire at the price they specify. Therefore in order to properly model the effect of the order book on trade within financial markets it is important that this feature is captured. To do this we allow traders to make offers at random prices for random quantities.

4

Despite its rich dynamics, the model is quite simple which is instrumental in obtaining clear-cut interpretations of the mechanisms driving the market dynamics. This will be particularly valuable when confronting our findings with a empirically observed phenomena. It is of course possible to formulate models which more accurately reproduce various features of the market behaviour e.g. Maslov (2000) which accurately captures many stylised facts observed within markets or papers such as Chiarella et al. (2006), Westerhoff and Dieci (2006) and Arthur et al. (1997) which use boundedly rational traders in an attempts to model the effect of different strategies employed within markets. Both of these approaches are important as they help us to understand how markets might operate and how the dynamics present could be explained. However, in our model we are attempting to understand the effect of the market mechanism on the market dynamics. It is therefore important to keep the model as simple as possible because adding even simple strategy to traders it becomes impossible to isolate the effect of the market mechanism from that of the trader behaviour.

The paper is organized as follows. The model is explained in detail in Section 2. Results are reported in Sections 3, 4 and 5. Section 6 concludes.

## 2    Model

This section sets out a model of a limit-order driven market with random order submission in which traders do not engage in strategic considerations or forward planning. Only a no-loss constraint on their limit orders is imposed. The model extends the zero-intelligent agent models by Gode and Sunder (1993) and Bollerslev and Domowitz (1992). The main innovations here are orders with price and quantity, continuous trading through market entry and exit of traders and a multi-unit order book.

*Traders.* The population of traders consists of two groups, buyers and sellers, who exchange an indivisible commodity/asset for money at discrete times. Each buyer entering the market is endowed with a common demand function which is represented by a set of discrete reservation prices $r_1^b \geq ... \geq r_m^b$. Each reservation price allows the trader to purchase a single unit of the commodity. The profit from purchasing one unit at price $p$ and matching it with the reservation price $r$ is $r - p$. Sellers are defined analogously with an initial supply function corresponding to the reservation prices $r_1^s \leq ... \leq r_n^s$ and profit $p - r$. A transaction removes both parties' reservation price and accordingly changes the individuals' supply or demand function. Each period, a fictitious, competitive equilibrium price can be defined through the current market supply and demand (i.e. the aggregate supply resp. demand

5

function of all traders present in the market).

*Order generation.* Traders submit orders by randomly drawing (with a uniform distribution) a price-quantity pair, $(p, q)$, from all feasible combinations. A pair $(p, q)$ is called feasible, if at price $p$ the trader has at least $q$ untraded units such no unit incurs a loss if a trade were executed at price $p$. The price $p$ is constrained by a strictly positive tick size and market minimum and maximum prices, $\underline{P}$ and $\overline{P}$, with $\underline{P} \leq r_1^s, r_m^b$ and $r_n^s, r_1^b \leq \overline{P}$. The quantity $q$ is restricted to integer values.

*Order book.* The market operates in discrete time, each time step being characterised by a single order submission of a randomly chosen trader (uniformly across all traders in the market).[1] An order book is maintained to which these orders are submitted and matched. All trade is performed through this book by executing any order that, upon submission, crosses the spread. Trade is executed at the value of the best price (the highest priced bid resp. lowest priced ask). The usual price/time priority applies and partial execution of orders is possible. Traded units are removed from the book and demand resp. supply function of the two traders are updated by removing the (highest resp. lowest) reservation prices to maximize each trader's payoff from the transaction. If the order does not result in a trade, it is added to the order book as a limit order. The submission of the new order removes a trader's previous order, whether or not the order is a limit order, to ensure that obligations can always be met. A cancellation process other than this is not modelled.

*Entry and exit.* Market entry and exit of traders is also modelled in a random fashion. Every time step, with probability $P_{in}$, a new trader enters the market and, with probability $P_{out}$, a randomly chosen trader (uniformly across all traders in the market) leaves the market. The new trader is a buyer with probability $P_{buyer}$ and will be endowed with a demand function as described above; otherwise it is a seller. With the exit of a trader all of his remaining reservation prices and outstanding orders, if any, are removed from the book. These parameters allow to control the average number of traders in the market, the ratio of buyers to sellers and the average time a trader stays in the market.

Let us address two possible critiques. First, the random order submission process generates a substantial amount of orders which will never (or with very small probability) be executed because they are too far away from the best quote. In this paper statistics are calculated after excluding those or-

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[1]This definition of time does not allow for a meaningful study of the intensity of order arrivals.

ders.[2] One could alternatively modify the random order submission process (see e.g. Farmer et al., 2005) but we refrain from doing so because it adds strategic behaviour. Second, buyers and sellers have identical opportunity costs. This is clearly not realistic if financial assets are traded because an unsuccessful seller is exposed to the risk associated to this asset while the unsuccessful buyer is not, this effect is observed in Lo et al. (2002) who document that sellers are more interested in immediacy than buyers as on average they place orders closer to the best quote. Though this cost is taken into account in most theoretical studies, we refrain from rectifying this issue to maintain the simplicity of the model (for examples of the effect of this difference see Biais et al., 1995; Ranaldo, 2004). Each trader is fully characterised by its residual demand resp. supply function.

*Parameter settings.* All simulations use the same set of parameters. The initial population of traders has size 100. Each trader entering the market is issued with 50 reservation prices: $100, 98, ..., 2$ for buyers and $0, 2, ..., 98$ for sellers. Market minimum resp. maximum prices are set to $\underline{P} = 0$ resp. $\overline{P} = 100$. Traders enter resp. leave the market with probability $P_{in} = P_{out} = 1\%$. The average number of traders in the market is therefore constant and equal to the size of the initial population. Each new trader has an equal probability of being a buyer or a seller, $P_{buyer} = P_{seller} = 0.5$. Statistics use time series data between iteration 100,000 and 2.1 million. This ensures that initial effects are washed out.

*Price dynamics.* A typical time series of the mid point of the bid-ask spread and the (instantaneous) competitive equilibrium price (i.e. the intersection point of the aggregated supply and demand functions) is provided in Figure 1.

The equilibrium price remains constant throughout long time periods and varies very little over the entire period while the market price, which centres around the equilibrium price, shows considerably higher variability. These dynamics imply that trade is very active and that mostly intramarginal (or marginal) units are traded. The latter is accentuated by traders always redeeming their most profitable units (i.e. the best reservation prices) first. These data also show the inertia of the limit order book market: the market and the equilibrium price do not coincide most of the time. Figure 1 documents a large band of market prices which vary between 40 and 65. In light of the wide range of trader's valuations (0 to 100), this feature should be interpreted as a local rather than absolute deviation of prices.

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[2]Empirical data often come with similar limitations, see, e.g. Biais et al. (1995).
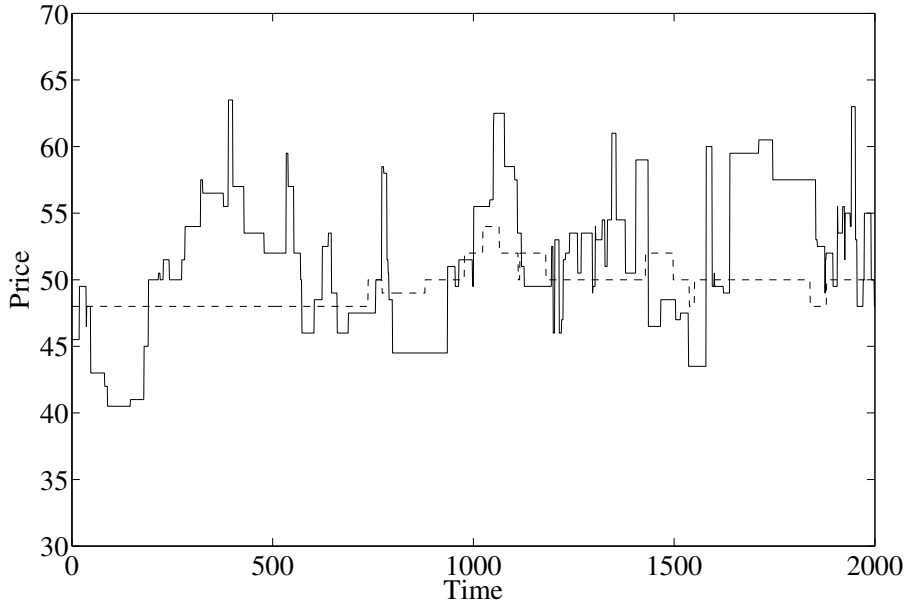
Figure 1: Market price (solid line) and equilibrium price (broken line) for 2,000 periods after the initial phase of length 100,000.

# 3 Shape of the order book

The shape and behaviour of the order book is the subject of many empirical studies, covering a variety of time series data and exchanges. In this section we compare related empirical results with the behaviour of the model.

## 3.1 Spreads and depths

In order to facilitate analysis we generate an 'average' order book composed of the time series means of the prices and quantities of the five best bid and ask quotes present within the order book. Summary statistics from this analysis are presented in Table 1. By only considering the first five bids and asks we reduce the huge amount of information present in the complete order book whilst keeping the most important. All trades occur at the tips and there is a sharply decreasing chance of execution with distance orders are placed behind the best quotes (Griffiths et al., 2000; Hollifield et al., 2004). It is also the area that is most important for trader decisions (Harris and Panchapagesan, 2005).

*Spreads.* The simulated bid-ask spread is found to be twice the width of either of the adjacent spreads in the order book (Table 1). This result is in

8

| Bid Prices | 146.4 | 143.1 | 141.0 | 139.5 | 138.28 |
| Offer Prices | 153.3 | 156.6 | 158.7 | 160.2 | 161.51 |
| Bid Quantities | 6.073 | 5.879 | 5.812 | 5.794 | 5.811 |
| Offer Quantities | 6.048 | 5.884 | 5.754 | 5.732 | 5.794 |

Table 1: Summary statistics from a simulation run of 20,000,000 time steps.

agreement with empirical findings Biais et al. (1995) for the Paris Bourse. Though it does differ from that of Ranaldo (2004) who find the spread to be about two ticks.

The remaining spreads in the order book are found to gradually decrease in size towards one tick as distance from the best quotes increases. Empirical evidence (Biais et al., 1995) suggests that spreads between adjacent orders were statistically equal. Other work (Hamao and Hasbrouck, 1995; Sandås, 2001) showed that internal spreads were very close to the minimum tick size. So in this aspect the model does not appear to completely capture the market dynamics.

*Depths.* The simulation results show that the depth available remains almost constant across the tip of the book. In line with our findings Sandås (2001) shows that the volume available at the best and second best quotes is equal. Biais et al. (1995) demonstrate that for the remainder of the book the depths available are constant.[3]

*Interpretation.* There are many different aspects of trader strategies that could contribute to this difference. One possibility is the order cancellation process. In this model order cancellation strategies were not considered though in real market data they are known to have a significant impact (nearly 10% of market actions are cancellations on the Paris Bourse (Biais et al., 1995)). In this model there are no cancellation orders and the chance of orders being matched in trade decreases as distance from the best quotes increases. As a result the density of orders will tend to increase as distance from the quotes increases. It is known that orders more than a few ticks from the best quote (Harris and Hasbrouck, 1996; Griffiths et al., 2000; Hollifield et al., 2004) are very unlikely to be executed and are often cancelled (many at the end of the day). If order cancellation and resubmission at more competitive prices were considered it is conceivable that the resulting price distribution would change to one which was more even with respect to price. This is only one possible suggestion, there are many other strategic aspects which will effect this distribution such as traders placing orders nearer the

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[3]For some stocks the depths at the best quote is actually slightly smaller, however, this is not true for all data analysed.

quote rather than uniformly to increase the chance of execution. Though in order to investigate these differences it would be necessary to employ a new model which considers the multitude of trader strategies.

## 3.2   Effect of the tick size

In this section the effect of tick size on market behaviour is examined. The empirical findings of Biais et al. (1995) showed that for stocks traded with tick sizes of 1FF the median number of ticks between adjacent orders other than the best quotes was one. Adjacent quotes were on average at adjacent prices and as a consequence there were no holes in the order book (similar to the results found by Hamao and Hasbrouck, 1995; Sandås, 2001). Though for stocks traded with a tick size of 0.1FF the median number of ticks between adjacent orders increased. The model accurately reproduces this result. When prices are restricted to integer values the median number of ticks between adjacent quotes is one and when the tick size is reduced to 0.1 the median number of ticks between orders increases. In both the empirical and model generated data the decrease in tick size leads to the formation of gaps in the order book indicating that this aspect of the market dynamics is probably as a result of having a system based on a discrete tick size and is not a direct result of trader strategies.

The results in Table 2 show that as tick size is reduced the volume offered at the best quotes also reduces. This result is in agreement with that of Goldstein and Kavajecz (2000) who found that the volume displayed at the best quotes of the NYSE reduced when the tick size was reduced. Goettler et al. (2005) point to a similar reduction in the spread when tick size was reduced. It is, however, harder to draw conclusions from our model along these lines as all the spread sizes round to the same value.[4]

| Tick Size | Spread in Ticks | Quantity Best Bid | Quantity Best Ask |
|-----------|-----------------|-------------------|-------------------|
| 10 | 1 | 6.96 | 6.96 |
| 1 | 7 | 6.19 | 6.19 |
| 0.1 | 69 | 6.14 | 6.12 |

Table 2: Effect of varying tick size.

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[4]The results also support the choice of tick size made for this model. It can be seen that as tick size reduces there is convergence in the size of the spread and the depth available at the spread.

# 4 Order submission dynamics

This section provides an analysis of the regularities in the flow of orders in the model. The order flow is a key theme in empirical studies of financial markets. Theoretical contributions often aim to explain the empirical regularities by modelling strategic behaviour which is rooted in information effects, trade-off between immediacy and price improvement, optimal limit order submission strategies and the optimality of different order types.

Our simulation study shows that several features found in empirical studies come about without any strategic consideration on the part of traders; they are the result of the market architecture. These include the conditional frequencies of order types, predictability of price movements and the effect of changing spread sizes. The results differ from the empirical findings in the absolute frequency of price-improving limit orders and market orders and the effect of depth on order submission.

Order types are classified as in Biais et al. (1995) except for cancellations and applications:

|  |  | Buy | Sell |
|---|---|---|---|
| **Market orders** | quantity greater than depth at best quote | B1 | S1 |
|  | quantity same as depth at best quote | B2 | S2 |
|  | quantity less than depth at best quote | B3 | S3 |
| **Limit orders** | within quotes | B4 | S4 |
|  | at quote | B5 | S5 |
|  | behind quote | B6 | S6 |

Table 3: Classification of orders in decreasing aggressiveness.

## 4.1 Conditional frequencies of order types

In financial markets the sequence of order arrivals exhibits positive correlations in the types of orders submitted (buy/sell, market/limit). Empirically Biais et al. (1995) demonstrates that under the order decomposition discussed above most types of orders are most likely to be preceded by the same type.

The conditional probabilities of order types, which describe the statistical likelihood of an order type following each other, are reported in Table 4. Highlighted in boldface is the maximum in each column. The type in the corresponding row is the most likely predecessor of each order type. These results can be compared with Table IV in Biais et al. (1995) (except for cancellations and applications).

|    | B1 | B2 | B3 | B4 | B5 | B6 | S1 | S2 | S3 | S4 | S5 | S6 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|
| B1 | **2.11** | 0.08 | 0.41 | 1.84 | 0.20 | 46.14 | 0.89 | **0.17** | 0.84 | 2.60 | 0.37 | 44.34 |
| B2 | 0.47 | 0.08 | 0.46 | 3.43 | 0.35 | 45.43 | 0.53 | 0.08 | 0.50 | 3.11 | 0.28 | 45.29 |
| B3 | 0.89 | **0.19** | **1.86** | 2.40 | 0.34 | 44.94 | 0.45 | 0.10 | 0.45 | 1.47 | 0.21 | 46.71 |
| B4 | 0.43 | 0.08 | 0.48 | **23.47** | 0.32 | 33.37 | 0.50 | 0.10 | 0.55 | 7.04 | 0.35 | 33.32 |
| B5 | 0.68 | 0.15 | 0.68 | 4.36 | **0.41** | 43.79 | 0.37 | 0.06 | 0.43 | 3.27 | 0.29 | 45.50 |
| B6 | 0.56 | 0.11 | 0.56 | 1.79 | 0.26 | **47.62** | 0.57 | 0.11 | 0.57 | 1.83 | 0.26 | 45.77 |
| S1 | 0.89 | 0.17 | 0.92 | 2.51 | 0.38 | 44.54 | **1.09** | 0.08 | 0.40 | 1.86 | 0.22 | 46.93 |
| S2 | 0.46 | 0.08 | 0.46 | 3.15 | 0.32 | 44.73 | 0.46 | 0.06 | 0.41 | 3.24 | 0.31 | 46.31 |
| S3 | 0.45 | 0.09 | 0.45 | 1.58 | 0.19 | 47.17 | 0.92 | 0.17 | **1.00** | 2.39 | 0.29 | 45.29 |
| S4 | 0.49 | 0.09 | 0.53 | 7.15 | 0.35 | 33.13 | 0.44 | 0.08 | 0.48 | **23.42** | 0.33 | 33.50 |
| S5 | 0.32 | 0.09 | 0.40 | 3.27 | 0.26 | 45.21 | 0.72 | 0.13 | 0.70 | 4.43 | **0.49** | 43.99 |
| S6 | 0.57 | 0.11 | 0.57 | 1.84 | 0.26 | 45.53 | 0.56 | 0.11 | 0.56 | 1.79 | 0.26 | **47.86** |
|    | 0.57 | 0.11 | 0.57 | 2.51 | 0.27 | 45.87 | 0.57 | 0.11 | 0.56 | 2.50 | 0.27 | 46.11 |

Table 4: Conditional probabilities (in %) of the order types defined in Table 3. Each entry in the main body denotes the probability of order types (columns) conditioned on the current order type (rows). The most likely predecessor of any type (columns) is highlighted. The unconditional probabilities of order types (columns) are given in the bottom row.

The most striking finding in Table 4 is that for ten of the twelve order types the maximum lies on the diagonal: events of a particular type are most likely to be preceded by another event of the *same* type. This finding is a near perfect match of the "diagonal rule". Indeed the fit to the diagonal is even better, as only the highest value (rather than the three highest values as in Biais et al.) is highlighted in bold face. The only exceptions are B2 and S2 (buy and sell at quantity same as depth at best quote).

In addition this effect is documented by Griffiths et al. (2000); Lo et al. (2002); Hall and Hautsch (2006) who all show that a market order increases the likelihood of other market orders from the same side of the market following it. Analytical this has been reproduced in several studies e.g. Seppi (1997). Initially studies hypothesised the cause of this regularity to be traders' strategic behaviour and/or information effects. For instance from analysing the NYSE, Hasbrouck (1991a) conjectures that positive correlations are observed as a result of the specialist trying to fulfil continuity requirements. Hamao and Hasbrouck (1995) however, later dismiss this explanation based on evidence from the Tokyo stock exchange, a pure limit order market, in which correlation is still present. This being the case Biais et al. (1995) put forward hypotheses pertaining to strategic order splitting, imitation behaviour and reactions to new (public) information. This is a view also taken by Huang and Stoll (1994) who argue that serial correlations can be explained by order splitting or the way in which information is incorporated into the market price. Hollifield et al. (2004) take a different view based on their analysis of order placement strategies on the Stockholm Stock Exchange, the cause of autocorrelation is the slow adjustment of the order book to changes in the value of the asset resulting in correlation in order types. This view is also shared by Goettler et al. (2005) who use results from an analytical model to argue that the correlation is due to corrections of mispricings.

As order submissions are governed by a random process, the regularities in the sequence of order types must be caused by the market mechanism and its interplay with individual demand and supply functions rather than by traders' strategic behaviour or information effects. The intuition for this remarkable result is closely tied to the explanations given by Hollifield et al. (2004) and Goettler et al. (2005): the market mechanism entails a sluggish adjustment of the order book in its role of appropriately reflecting the aggregate demand and supply which drives the regularity in the order flow.

Details about how the mid quote is able to move away from the (theoretical) equilibrium price, if the state of the order book does not adequately reflect the residual market demand and supply is provided in Section 5. If this occurs the asset is effectively mispriced as the mid quote of the order

13

book does not agree with the equilibrium price. This apparent mispricing has a similar effect to a change in the consensus value as documented in Goettler et al. (2005). The consensus value of the traders will be the equilibrium price even if the order book does not reflect this. In this situation Goettler et al. (2005) argue that a sequence of orders should be observed which gradually correct the mispricing such as the many small market orders observed by Biais et al. (1995) and Ranaldo (2004) amongst others and as Hollifield et al. (2004) state unless mispricings are immediately incorporated into the book there will be autocorrelation in the orders. Cordella and Foucault (1999) use a similar argument to explain why sequences of limit orders may be observed. Suppose the equilibrium price is between the best quotes and the spread is wide. Then traders will place limit orders which gradually close the spread.

In the case of this model, the no loss price/quantity constraint placed upon trades means that the mechanisms described by Cordella and Foucault (1999) for limit orders and Goettler et al. (2005) and Hollifield et al. (2004) for market orders will also be in effect here. The triangular nature of this distribution means that there is less chance of placing an order at the far side of the spread than the near side and quantities are on average smaller. As a consequence sequences of small market orders slowly correcting an imbalance are more likely than correcting it quickly with large orders and limit orders will tend to gradually eat away at the spread from there own side of the market. In essence although the market will tend to revert towards the equilibrium this reversion will only happen slowly and each order that corrects imbalances will only contribute a small part. The current market conditions which make a particular type of order more likely have high persistence even following orders of that type. The only exceptions to the observed pattern are buy and sell market orders of the same size as the best quote. These orders tend to follow small market orders from the same side of the market or large market orders from the opposite side. The reason for this is again connected with the nature of the spread. If a market order removes all the depth at the best quote then the new best quote is further from the other side of the market, as a consequence it will on average have a higher depth available where as the average quantity of a market order reaching it will be smaller, hence it is unlikely that sequence of these types of orders will be observed.

This result shows that the diagonal rule can emerge from the interplay of the order book and the individual demand/supply functions without any strategic behaviour or information effects.

## 4.2 Absolute frequencies of order types

The importance of trader strategy on market behaviour is revealed in the absolute frequencies of orders. Comparing the distributions of order types between empirical and model data show that real traders make strategic tradeoffs between immediacy and price improvement. Through doing this they are able to avoid losses from needlessly walking up the order book or closing the spread.

A significant impact of strategic behaviour is in the submission of orders behind the best quotes. In real markets relatively few orders submitted behind the best quotes are executed (Griffiths et al., 2000) and as a result relatively few limit orders are placed in this area. Zero-intelligence traders, however, are not capable of making this decision therefore to conduct a meaningful analysis of the absolute frequencies in the simulated model the empirical data requires that the time series of submitted orders be adjusted for the too high number of orders far behind the best quotes. This is done by removing orders more than four ticks behind the best quote on the corresponding side of the market. The adjustment keeps the ratios of order frequencies within the first 5 types unchanged (excluding orders behind the best quote, type B6/S6).

### 4.2.1 Market vs. limit orders

This section examines the total quantities of market and limit orders submitted by traders. The absolute frequencies of order types in the model (after adjustment) and, for reference, the corresponding data from Biais et al. (1995, Table III) are reported in Table 5. The data from Biais et al. are adjusted for cancellations and applications.

| Order type | | Biais et al. (1995) | Total | Model | Total |
|---|---|---|---|---|---|
| | | Average [Buy/Sell] | | | |
| Market | B1/S1 | 3.50 [2.71/4.29] | 6.04 | 5.36 [5.38/5.33] | 6.39 |
| | B2/S2 | 2.54 [2.37/2.71] | | 1.03 [1.00/1.06] | |
| Limit | B3/S3 | 20.62 [14.12/27.11] | 30.40 | 5.30 [5.32/5.28] | 28.88 |
| | B4/S4 | 9.78 [10.17/9.38] | | 23.58 [23.63/23.53] | |
| | B5/S5 | 5.43 [5.99/4.86] | 13.57 | 2.49 [2.50/2.49] | 14.76 |
| | B6/S6 | 8.14 [8.36/7.91] | | 12.27 [12.29/12.25] | |

Table 5: Absolute frequencies of the twelve order types (in %). Orders which are 4 or more ticks behind the best quote at the time of submission are removed from these statistics.

The ratios of orders submitted reveal important details of trader behaviour. Generally the frequencies of the particular order types differ substantially between the random order submission model and empirical data as documented in Table 5. In contrast, the cumulative frequency of market orders of size larger than or equal to the depth at the best quote (i.e. the sum of frequencies of types B1/S1 and B2/S2) is almost identical. The same holds for the sum of the frequencies of small market orders and limit orders within the spread (i.e. the sum of frequencies of types B3/S3 and B4/S4). The systematic deviations from the empirical observations points toward the presence of two important tradeoffs which must be related to the traders strategic/optimal behaviour.

*Aggressiveness.* Market orders at the exact depth of the best quote occur far too often in comparison to those generated by a random order submission process (Ranaldo, 2004; Hollifield et al., 2004). Since aggressive market orders (types B1/S1 and B2/S2) are observed with the same frequency, it can be conjectured that traders who want to buy a large quantity often prefer to take all orders at the best quote and wait for the book to fill rather than going immediately for more expensive quotes. Glosten (1994) highlights a possible reason for this by demonstrating that small trades are profitable as the consensus value of the asset only moves a small amount subsequent to trade, where as large trades with their relatively higher cost per share result in potentially large shifts in the consensus value making them unprofitable (in the absence of strong information).

*Price improvement.* The simulated model has a substantial amount of limit orders that improve the price by more than one tick and very often are placed beyond the mid point of the spread. These limit orders obtain higher priority than existing ones but effectively pay most of the cost for crossing the spread whilst still potentially incurring a non-execution cost (Cohen et al., 1981, explain why limit orders should not be placed arbitrarily close to the opposite side of the market). Given that there is a cost to crossing the spread (Huang and Stoll, 1994), the empirical observations show that it is more profitable for traders to either make a small market order that executes immediately (Cohen et al., 1981) or to make a limit order which just improves on the best quote by a small amount (Harris and Hasbrouck, 1996, calculate that these are the best performing).

### 4.2.2 Dependence on size of spread

This section investigates the effect of the size of the spread on the absolute frequencies of orders submitted. Table 6 summarizes the simulated data. The presentation follows the empirical benchmark by Biais et al. (1995, Table V).

16

Most notably we can see that when spreads are large more limit orders are placed within the spread and less market orders are submitted. This result is in strong agreement with the results obtained by (Biais et al., 1995; Foucault, 1999; Hollifield et al., 2004).

|  | B1-B3 | B4 | B5 | B6 | S1-S3 | S4 | S5 | S6 |
|---|---|---|---|---|---|---|---|---|
| Small Spread | 20.78 | 10.79 | 3.03 | 15.47 | 20.80 | 10.75 | 3.00 | 15.39 |
| Large Spread | 5.00 | 30.23 | 2.62 | 12.19 | 5.02 | 30.14 | 2.61 | 12.18 |

Table 6: Actions relative to spread.

Biais et al. (1995, page 1675) conjecture that the reason for increase limit order submission when spreads are wide is due to competition amongst traders to supply liquidity and uses this as an argument as to why the cause can not be due to randomness. Harris and Hasbrouck (1996) also point out a strategic reason for limit orders being more heavily used when spreads are wide as they allow traders to avoids paying the cost of crossing the spread. The results from this model, however, suggest that the increased limit order submission may be obtained simply as a result of the spread being wider. By increasing the spread there is a larger area in which limit orders can fall, therefore increasing their likeliness. It is not necessary that traders make strategic decisions to obtain this effect.

Naturally as a consequence of more limit orders being placed within the spread the spread shrinks. Similarly when the spread is smaller, more trade occurs (market orders) resulting in a widening of the spread (Lee et al., 1993). As Hall and Hautsch (2006) show, as terms of trade worsen traders willingness to trade reduces and the longer limit orders tend to stay unexecuted (Lo et al., 2002). Cohen et al. (1981) postulate that there is an equilibrium spread which markets tend to move towards. The randomness in the market (prices, order of order submission) leads to the spread moving away from this equilibrium whilst the market forces counteract this movement. As a result we see the spread alternating between a large and small size with more trade occurring when the spread is small (cf. Foucault et al., 2005).

The effect of the depth available at the best quote on absolute frequencies was also examined. It was found, however, that changes relative to the depth were small in magnitude indicating that both the effects and theoretical predictions detailed by Hamao and Hasbrouck (1995); Parlour (1998); Sandås (2001) amongst others may be due to traders' strategic actions rather than the market mechanism.

# 5 Predictability of price movements

Both theoretical and empirical results suggest that the order book may be an indicator of likely future price movements. Empirical work by Hasbrouck (1991b), Huang and Stoll (1994), Harris and Panchapagesan (2005) has shown that the amount of order book information available to traders (and in the case of the NYSE the extra information available to the specialist) may provide a significant advantage to traders in understanding future movements of the market. Harris and Panchapagesan (2005) in particular suggest that the more information a trader has about the order book the more accurate a prediction of future behaviour they can make. On the NYSE specialists have access to the complete order book whereas other traders only know the best quotes or at best have aggregated statistics allowing specialists to trade profitably within the market even under the conditions imposed by their obligations to the exchange. Hall and Hautsch (2006) suggest that it is the tip of the order book which is particularly important in determining the future behaviour of traders and the market. Theoretical work (Parlour, 1998) demonstrates that the state of the order book and in particular the relative volumes offered at the best quotes has a strong impact on the behaviour of traders i.e. whether traders submit limit or market orders and hence whether prices change. In general, as Sandås (2001) points out, for most stocks the market order flow distribution strongly depends on state variables such as the past trading volume.

## 5.1 Analysis

We devise four measures of the order book state employing different amounts of information. These measures based on propositions put forward in the theoretical and empirical work mentioned above. We find the correlation of these measures with price movements at different time large in order to identify how effectively they predict price behaviour. If we define $q_n^b$ (respectively $q_n^s$) to be the quantity available at the $n$th buy (sell) quote, $p_n^b$ (respectively $p_n^s$) to be the price of the $n$th buy (sell) quote, $p_{mid} := (p_1^b + p_1^s)/2$ to be the mid price, and $d_{mid} := 1 + max(p_{mid} - p_5^b, p_5^s - p_{mid})$ (used to scale distances from the mid quote) then the four measures are defined as follows:

(i) $q_1^b - q_1^s$

(ii) $\sum_{n=1}^{5} q_n^b - \sum_{n=1}^{5} q_n^s$

(iii) $(\sum_{p=1}^{5} q_n^b[d_{mid} - (p_{mid} - p_n^b)]) - (\sum_{n=1}^{5} q_n^s[d_{mid} - (p_n^s - p_{mid})])$

(iv) $(\sum_{n=1}^{5} q_n^b/[p_1^b - p_5^b]) - (\sum_{n=1}^{5} q_n^s/[p_5^s - p_1^s])$

The first measure provides a baseline simply looking at the difference in quantities available at the best quotes. The second expands on this idea

based on the types of information sold by the NYSE, i.e. aggregate order book information, the difference in quantities aggregated over the first 5 quotes on either side of the book. The third measure is a rough approximation to the type of information specialists have available in the NYSE, i.e. they can see both quantities and prices. This measure simply weights the relative significance of quantities by their distance from the mid-price, i.e. quotes further from the mid-price and less likely to be traded are less heavily weighted. The fourth measure is based on the idea presented in Glosten (1994) and discussed in Biais et al. (1995), which measures the relative slopes of the order books.

## 5.2   Results



(a) Measure (i)

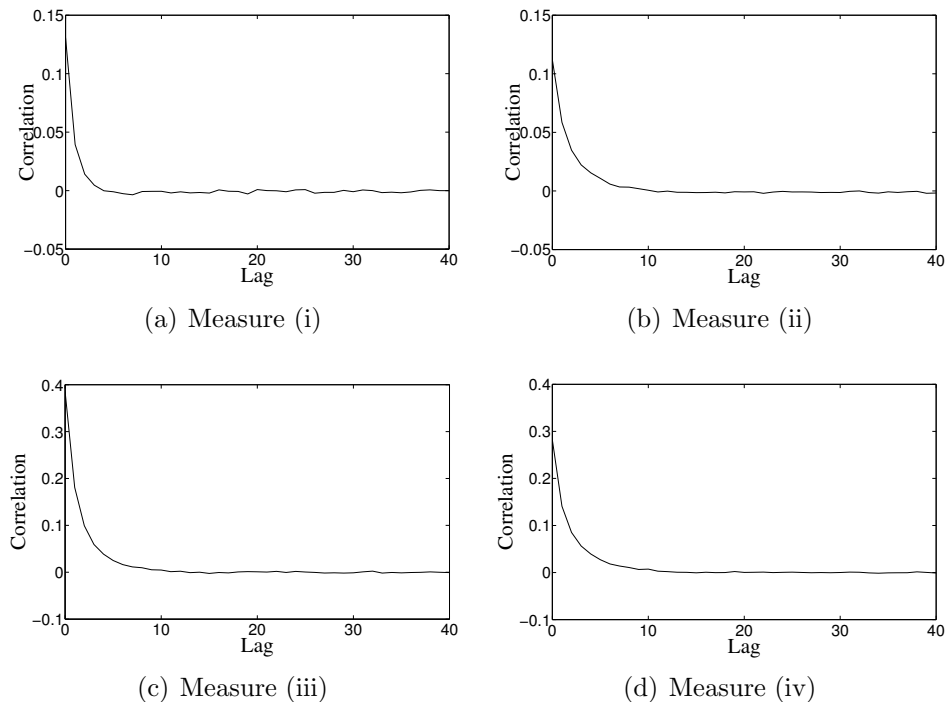(b) Measure (ii)

(c) Measure (iii)

(d) Measure (iv)

Figure 2: Correlation between price movements and the four measures defined in Section 5.1 up to a lag of 40 time steps.

The results (Figures 2(a)-2(d)) show the correlations obtained with the four measures described above. To a lesser or greater degree all measures exhibit positive correlation between price movements and the state of the order book indicating that price movements tend to be towards the weaker side of the order book. This is in agreement with Hall and Hautsch (2006),

Huang and Stoll (1994) and Harris and Panchapagesan (2005) who demonstrate that short term market behaviour may to some extent be predicted from the order book shape.

A small correlation is observed in the case of the first measure which is in close agreement with both empirical and theoretical findings. Empirically Hall and Hautsch (2006) show that a decrease in volume offered decreases the relative buy pressure from that side of the market encourage price movements in the opposite direction and Huang and Stoll (1994) show that when the available depth at the bid exceeds that at the ask price returns tend to be positive. Analytically Parlour's model predicts that when one side of the market has more depth at the best quote than the other it is more likely that market orders will be submitted by traders on the heavier side and limit orders by traders on the lighter side as a consequence their is a pressure for a price movement away from the heavier side. (Note in Parlour's model this movement is impossible due to it only considering two prices.) This result is extended in the second measure, in particular it agrees with the results of Harris and Panchapagesan (2005) who show that price increases are more likely when the buy side of the order book is heavier.

The correlations obtained with depth information alone are relatively weak, however, when price information is also included the correlations are increased. Measure three demonstrates that detailed order book information may be exploited by traders to make profits in line with the results of Harris and Panchapagesan (2005). In particular measure three agrees with the results of Hall and Hautsch (2006) who show that a decrease in the slope of the order book decreases the pressure from that side and so makes price movements towards that direction more likely.

Measure four indicates that price movements will be towards the flatter side of the order book and away from the steeper side. This result is in agreement with the analytical work of Glosten (1994) and empirical work by Biais et al. (1995).

## 5.3   Interpretation

All four measures were constructed so that a positive value indicates that the buy side of the book is stronger than the sell side of the book and so consequently the positive correlation indicates movements towards the weaker side of the book (an increase of price when the buy side is heavier). In order to understand why this occurs it is important to note that order density increases with distance from the equilibrium price this is because (a) the more competitively priced an order is the more likely it will be matched in trade and removed and (b) higher quantities may be offered at less competitive

prices. As a consequence of this as the mid-price (and spread) move away from the equilibrium price the tip of the book in the direction moved will be packed (relatively) more densely both in terms of shrinking spreads and increasing depths than the opposite side of the order book.

In all cases the correlation rapidly approach zero at longer lags. This is because as time passes it is more likely that an order will have been submitted redressing the original imbalance, which is in agreement with the findings of Harris and Panchapagesan (2005) who show that the order book provides much better predictions at short lags than long.

# 6    Conclusion

By reproducing stylised facts observed in financial market data with a model completely void of rational traders (or intelligence) our paper shows that certain market behaviours are a result of the limit order book rather than traders' strategic behaviour. It is shown that the limit order book has a significant effect on the time series of orders submitted to the market in particular affecting the the conditional probabilities of orders. The book also appears to govern various aspects of the shape of the order book and the size of the spread. In comparison the considerations of traders with regard to aggressiveness and price improvement have significant impacts on the frequencies of the different order types submitted. Price behaviour is found to exhibit some predictability in the short term, based on the observed order book state, in line with findings from analytical studies.

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