Agent-Based Modelling, a New Kind of Research

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Abstract

We discuss the use of Agent-based Modelling for the development and testing of theories about emergent social phenomena in marketing and the social sciences in general. It addresses both theoretical aspects about the types of phenomena that are suitably addressed with this approach and practical guidelines to help plan and structure the development of a theory about the causes of such a phenomenon in conjunction with a matching ABM. We argue that research about complex social phenomena is still largely fundamental research and therefore an iterative and cyclical development process of both theory and model is to be expected. To better anticipate and manage this process, we provide theoretical and practical guidelines. These may help to identify and structure the domain of candidate explanations for a social phenomenon, and furthermore assist the process of model implementation and subsequent development. The main goal of this paper was to make research on complex social systems more accessible and help anticipate and structure the research process.

Keywords: Agent-Based Modelling, Model Development, Research Process, Modelling Cycle, Methodology

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1. Introduction

Computational models and especially Agent-based Models (ABMs) are a comparatively new addition to our analytic toolkit in the social sciences generally and in marketing in particular. They open a new level of analytic analysis in between established modelling approaches that describe either an individual's behaviour and decision making, or the relationships between aggregated measures that characterise a population as a whole. The tools used in those traditional approaches are usually statistic models such as regression or structural equation models, or mathematic methods including differential equations and analysis of equilibria and limiting behaviour.

ABM and related computational tools do not replace these models, but add a new dimension to the range of issues that we can address with formal means: with ABM we are capable of formalising how individual actions and decision making bring about aggregate characteristics of a population, which is especially interesting when the aggregate is brought about through the individuals' interactions. ABM takes into account that individuals generally do not exist in isolation, but are interdependent, mutually affecting each other through their action and interactions, directly and indirectly, intentionally or unintentionally. This kind of thinking challenges reductionist notions that claim we can exhaustively understand the world by dissecting it into smaller and smaller pieces. Instead, it proclaims that structure matters!

Much theory has been developed about the functioning of systems in general (von Bertalanffy, 1968; Holland, 1992; Luhmann, 1997), but now, ABM provides a way to formalise such theories, explore the consequences of our assumptions through computational experiments and ultimately develop tests about the validity of our theories through comparison with empirical data.

Compared to traditional tools of analysis, the use of ABM is still in its infancy, especially in the field of marketing. But there are signs of increasing interest (e.g. Delre et al., 2007; Watts & Dodds, 2007; Delre et al., 2010; Rand & Rust, 2011). The skills to build computational models are rare among faculty and students, and few are familiar with ABM and associated theories about interdependent, complex systems. Consequently little effort is devoted to identifying and describing suitable problems that could now be addressed with these tools. At the same time, the method itself is still under development, which means that there are not yet many established procedures to assess the validity of a model, and much of the effort that goes into the formal implementation of a theory as ABM needs to be devoted to methodological considerations. The learning curve may be steep, but existing examples and applications of this method are promising and suggest growing importance in the future.

This article attempts to reduce the inhibition threshold that may be associated with a new tool like ABM, describing its theoretic background and practical considerations regarding the model development process. It is however not an introduction to programming. There are many good text books available that address that issue (see e.g. Gilbert, 2008; Railsback & Grimm, 2011). This article pursues three goals:

- Help the reader identify problems that are suitably addressed with ABM,
- Present guidelines for the development of theories about systems of interdependent actors,
- Showcase the development process of an ABM, focussing especially the importance of social mechanisms, modular programming and iterative development of model and theory

Section 2 will address theoretical aspects concerning ABM, starting with a discussion of the purposes of modelling and a brief description of the key features of ABM in particular. This is followed by the presentation of the framework of analytic sociology that helps us formulate and formalise theories about the causes of emergent social phenomena. This section will then conclude with a review of core aspects of complex systems thinking, highlighting concepts that make this line of thought notably different from traditional modelling approaches.

Section 3 presents a selection of practical guidelines for the development of ABMs, illustrated by examples from a recent ABM research project. These guidelines may help structure the research on emergent phenomena and better anticipate and address some of challenges that come with this new kind of research. Concretely, this section will discuss the identification of social mechanisms and their combination to a theory about emergent phenomena, as well as their implementation in individual software modules. Furthermore it will outline the process that is to be expected in developing an ABM.

Due to the nature of the method, the development process is most likely iterative and consecutive revisions of the model are to be anticipated. Such revisions are a natural part of the process of model implementation and reflect the learning process and improved understanding of the interdependencies between various model assumptions. In Sec. 4 the insights from this paper will be summarised and discussed.

2. Models - Understanding Through Analogy

Statistical and mathematical models are the predominant method to formally express theories used in marketing and other fields of social science. Especially inference statistical models, with their capability to formally test hypotheses on the basis of empirical data are the de-facto standard for scientific enquiry.

While these are likely the first examples to come to mind, the term "model" is much wider. Generally, models are simplifications of the real world, which are less detailed and/or less complex than the original (Gilbert & Troitzsch, 2005). They can serve many different purposes, including explanation, prediction, guidance to data collection, discovery of new questions, training, communication, etc. (see e.g. Epstein, 2008). It is due to their simplifications, that models are more easily accessible than the original; either in the sense that they allow us to do something with them that is not possible with the original or in the sense that they are easier to understand, highlighting only the most essential aspects without distracting details. Even though models are just analogies, not the real thing, understanding a model can help us better understand the original.

The same reasoning applies to computational models. They constitute an implementation of a theory about the processes that drive a system in reality. If the model is valid, in the sense that is reproduces features of the real system to a satisfactory degree, it is appropriate to assume that insights gained from a better understanding of the model apply to the original system by analogy. Moreover, computational models typically have the advantage that they can be monitored in great detail and they are amenable to a vast variety of manipulations. This is especially beneficial in the study of social systems where manipulations and experiments are often infeasible in the original system, due to practical, ethical or technological limitations. Computer models enable us to run *controlled experiments* on an entire system, which is the basis to identifying causal mechanisms that drive its development.

A computational model can be run repeatedly, and it is possible to systematically manipulate the model's conditions and monitor the resulting changes in the model's outcomes. Model development can be tracked using various measures and statistics, and can be analogs to values observed in the real world. In short we can do systematic experiments on our computational models, understand them in much greater detail than the original system, learn about the range of behaviour and results that are possible under our model assumptions and transfer these insight to the original system by analogy.

Considering that social systems are naturally path-dependent, computational models have another advantage: in the real world we have in effect a sample size $n = 1$, the history that happened to occur. Computer models can be run over and over again, to study the effects of chance and probability, but also the consequences of slight variation of parameters and starting conditions, asking "what if?" questions. This allows us to explore what could have happened, producing alternative histories that could have developed from the same principles, thereby giving us a sense of the robustness of the system, a measure of the range of possible developments and also a chance to learn about critical turning points in a system's development.

Computational models can be considered computer assisted-thought experiments (Di Paolo et al.). According to Kuhn (1977, p. 261) "thought experiments give the scientist access to information which is simultaneously at hand and yet somehow inaccessible to him". With their supreme logical capabilities, computers can help us deduce consequences of the set of assumptions - our theory - even when this deductive process requires thousands or millions of computations. For the computer, any simulation is merely a deterministic, recursive function depending only on a set of exogenous parameters and starting conditions (Leombruni & Richiardi, 2005; Borrill & Tesfatsion, 2010). The model specifies the rules of how the state of the system changes from step to step, and the computer calculates the system's development over time. Essentially, the computer provides numerical solutions to highly complex, non-linear mathematical problems. When our own logical capabilities reach their limits, we can use computers to do the chores for us, provided that that theory is presented in a way that is accessible to a computer. Bedau (1999) therefore refers to computers as a *philosophical crutch.*

Such thought experiments can help us clarify and better understand our own thinking, especially concerning theories about the development of complex systems comprised of autonomous, yet interdependent and somewhat intelligent entities. These systems can behave in unintuitive ways, partially because the interdependencies are so manyfold, and partially because we still lack training and an established body of literature dealing with such systems.

In addition to the logical deduction of consequences in our theories the formalisation of a theory in computer code also forces us to specify all assumptions of our theories in detail, covering the relevant properties, activities and interactions of the entities of interest, but also the crucial aspect of the context and environment that these entities are located in. It is part of this process to reveal gaps and imprecisions in our theories. Therefore the implementation of a model should be considered as part of the process of theory development.

Due to the fact that ABM models entire systems, not individual hypotheses, the development process of a systems theory has a peculiar structure. Not unlike Lakatos' (1970) description of research programmes, the theories implemented in computer models consist of a *hard core* of theoretical assumptions and a collection of *auxiliary hypotheses* (Cioffi-Revilla, 2010). If a computational model fails to reproduce empirical patterns, the first conclusion to be drawn from this is that the *set of assumptions as a whole* is inconsistent with the observations. Instead of discarding the entire model/theory, systematic analysis of the model constituents should then be used to improve the model's performance. This process is different to conventions for handling statistical models, as these usually test one well-specified hypothesis, not a set of assumptions and combination of hypotheses.

Using models as analogy to better understand another system comes with the obvious drawback that we are not studying the original system. Consequently, there may be several competing theories, and potentially a variety of possible implementations for each of them, that could explain the real system. So, while a model may well provide us with *an* explanation, we need to be aware that the model alone does not provide the means to guarantee that this is the only or true explanation. Epstein (2006) refers to this as *explanatory candidacy*. With formalised models we can compare these theories and assess their validity. Depending on the availability of data that can be compared with the model's outcomes, it may be possible to use for validation quantitative methods such as statistical tests or more qualitative criteria to assess the degree of congruence between the model and real observations (Marks, 2007).

2.1. Agent-Based Models

The focus of this article is on the conceptualisation of Agent-based Models (ABMs), therefore only a brief summary of the core aspects of this computational modelling technique will be provided here. More comprehensive introductions are readily available, see e.g. Gilbert (2008); Railsback & Grimm (2011); Van Dam et al. (2013) or Edmonds & Meyer (2013).

ABM developed as a sub-field of multi-agent systems in computer science. From the early 1990s on, advances in software programming and hardware allowed more and more programmers to build software agents in the form of self-contained programs that determine their own actions based on inputs from their operating environment (Huhns & Singh, 1998). Such agents are still used today, for example to collect information on web-pages on the internet. However, the most important development was that within their very specific and limited field of operation, these agents became in a limited sense *autonomous*.

It was the prospect of modelling the interactions of many autonomous individuals that strongly increased the interest in simulation as a tool for the social sciences (Gilbert & Troitzsch, 2005). In these models complex aggregate behaviour arises not through central coordination, but directly, *bottom-up* from actions and interactions of actors at the micro level (Marks, 2007).

From a computational perspective an agent consists only of a set of logical rules of behaviour and a list of internal states, representing for example its memory, mood or capabilities. All the agent does is to collect input about its current situation, both internal and external, and then match this input to the set of rules to deduce its reaction to the situation. For the understanding of social systems it is important that whole populations of such agents can be social in that they interact and influence each other when they are joined in a computer simulation. So we can study in great detail the development of their interactions over time.

Generally, such agents have limited, mostly local capacities to perceive their environment. They may be heterogenous regarding their capabilities or objectives and they may have means of interaction and communication. Metaphors such as beliefs, intentions, desires or even emotions are used frequently to describe the agents' internal states. Wooldridge & Jennings (1995) outline the typical properties of computer agents:

Autonomy: Agents control their own actions as well as their internal state. In particular the user does not interfere with their decision making, after they specified its rules.

Social Ability: Agents interact with other agents, on the basis of a common language or actions.

Reactivity: Agents are able to perceive their environment, including other agents, and they are able to react on the basis of these perceptions.

Proactivity: In addition to mere reactions to their environment, agents are also able to take initiative, engaging in goal directed behaviour.

ABMs highlight the importance of the agents' *interactions*, exploring how they jointly generate social phenomena, analogously to the way these phenomena are brought about in real life. As Epstein & Axtell (1996) already observed: *simple entities, interacting through simple, local rules can produce very complicated behaviour.*

The models necessarily include relevant aspects of the agents' environment, to provide the context for the agents' interactions. This environment can be physical or abstract, reproduce for example a geographic landscape or a social network. It can also contain passive agents, such as objects or resources that the active agents interact with. In some simulations the agents' locations are relevant, and they may be able to move through space, while others may choose to omit such a feature.

2.2. Modelling Causal Mechanisms

In contrast to many mathematical models, computational models need not be so strongly simplified to become analytically tractable. Abstractions and reductions of complexity are therefore not primarily a technical necessity but a deliberate theoretical choice. The outstanding conceptual advantage of ABMs is that their most basic units are autonomously acting agents that are represented with "*one-to-one correspondence*" (Gilbert, 2008): the modelled agents can correspond with individuals (or organisations) in the real world and their actions and interactions can likewise correspond to the actions and interactions between the real world actors. Through simulation of the actual activities, computational models can become analogues for the real system and allow us to study the causal effects that bring about change in a system governed by the modelled activities.

The type of explanation provided through ABM is different from statistical modelling. Statistical models explain the co-variation of variables, identifying parameters that maximise the fit of a certain statistical model structure to the co-variation in the data. However, co-variation is not causation. Only through controlled experiments that are usually independent of the statistical model can we get an impression of the causal processes that bring about the co-variation of variables. Statistical models themselves do not provide an explanation of why and how the observed variables correlate in the observed way.

ABMs however are capable of capturing the causal mechanisms that bring about an aggregate pattern from the bottom-up, modelling explicitly the entities and activities that - according to the theory - cause the phenomenon under investigation. In this sense ABM provides causal explanation of how the modelled system works, and then by analogy - provided there is a sufficiently close fit with empirical observations - these insights can be transferred to the real system. This particular understanding of the term "explanation" is described by Herbert Simon (1968):

> To "explain" an empirical regularity is to discover a set of simple mechanisms that would produce the former in any system governed by the latter. (p. 44).

Mechanisms are the basis of explanation in the physical sciences but they are not so well established in the social sciences, marketing and economics. However, there are signs of change. Recent developments in the field of Analytical Sociology (AS) focus on the way large scale social order emerges from social mechanisms driving micro actions and interactions (Hedström & Bearman, 2009), and they rely on computer simulations and especially ABMs.

Social phenomena in the sense of AS refer to properties or patterns observed in a group of individuals. The conceptual basis for AS is that the social and the individual mutually influence each other, but ultimately social phenomena are *caused* by the activities of individuals and need therefore be *explained* in these terms. All causes of social phenomena lie in the individuals and their activities, because only individuals with their actions have *causal powers*.

According to the AS framework the core of a causal explanation of any social phenomenon are social mechanisms in the following sense:

> Mechanisms can be said to consist of *entities* (with their properties) and the *activities* that these entities engage in, either by themselves or in concert with other entities. These activities bring about change, and the type of change brought about depends upon the properties of the entities and the way in which they are linked to one another. A social mechanism, as here defined, describes the constellation of entities and activities that are organised such that they regularly bring about a particular type of outcome. We explain an observed phenomenon by referring to the social mechanisms by which such a phenomenon is regularly brought about. (Hedström, 2005, p. 25, emphasis in original)

Such mechanisms are often left implicit in our theories, models and explanations, especially if we are focusing on the behaviour of variables rather than actors and events (Van de Ven & Engleman, 2004; Buttriss & Wilkinson, 2006).

ABMs offer a way to model the constellations of entities with their properties and interactions that bring about a social phenomenon. They are tools to capture the essence of a social phenomenon through its driving social mechanisms. As such ABMs offer a middle ground between "thick" and "thin" descriptions (McKelvey, 2004). At one extreme, thick descriptions are the result of in-depth case studies of actual histories, which reveal the complex causal processes involved but cannot be easily generalised. While at the other extreme there are thin descriptions that result from sample survey-type research. This research is more generalisable but abstracts away from any examination of the mechanisms, processes, events or choices by which different types of variables are interrelated and affect outcomes. ABMs abstract away from the concrete instance of a social phenomenon, but it maintains a causal explanation of the social mechanisms that bring about the phenomenon. This is a new way of conducting research that may help shed light on the understanding of complex social systems.

2.3. Theory of Emergence in Complex Systems

ABM is a very flexible tool to simulate the interactions of numerous interdependent entities. These models can represent social, biological or physical systems, or any combination thereof. Regardless, the advent of such systems-oriented thinking long precedes the development of ABM. Scientists have been theorising and investigating the properties and characteristics of interdependent systems with other means from the beginning of the 1960s (for a historic review see e.g. Sawyer, 2005). From these efforts gradually developed a field of research commonly referred to as the study of *complex adaptive systems* (CAS).

CAS research has been conducted with a multitude of methods and it continues to be an active field of scientific inquiry. CAS thinking laid the foundations for ABM, and as ABM gains popularity, its applications feed back into the CAS literature, so both areas are now highly interrelated and their boundaries are sometimes hard to define.

CAS research developed a range of concepts that may be helpful and important for the analysis and understanding of ABMs. These concepts include adaptivity, emergence, path-dependence, as well as attractor states and resilience. A comprehensive discussion of all the aspects and variants of complexity theory is beyond the scope of this article, but there is a range of introductory and advanced books available that focus on various dimensions of the research available (see e.g. Waldrop, 1992; Sawyer, 2005; Miller & Page, 2007; Mitchell, 2009; Edmonds & Meyer, 2013).

A *system*, as defined by Viscek (2002), is an entity that can be analysed on multiple levels, such as a micro (unit, individual) level and a macro (aggregate, social) level. The crucial characteristic of *complex* systems is that the laws which describe their behaviour on one level are *qualitatively different* from the behaviour on other levels. These qualitative differences stem from interactions and interdependencies between the entities on the micro level. Systems are referred to as *adaptive* systems, if micro entities can adapt their behaviour to their environment. Directly or indirectly this adaptive behaviour may include responding to aggregate states of the system that the entities shape themselves. CAS are characterised by bottom-up as well as top-down feedback processes. Opposed to social and biological systems (e.g. human societies, social insects or eco-systems), physical-material and chemical systems can be complex (e.g. hurricanes or turbulent rivers), but generally lack the feature of adaptivity.

The phenomenon that system level behaviour is qualitatively different from the behaviour at the entities' level, even though the aggregate behaviour is brought about exclusively through the individuals' actions and interactions, is referred to as *emergence*. Often not intuitively and sometimes quite surprisingly, complex systems exhibit pronounced changes in behaviour as we shift our perspective from one level of aggregation to another (Miller & Page, 2007; Mitchell, 2009). On the system level, we speak of aggregate patterns, spontaneous order or self-organisation - all these emergent phenomena are brought about *not* by a central coordinator but through local interaction between the entities in the system.

CAS develop dynamically and evolve over time. As a consequence their current and future range of development are largely determined by the states, events or decisions made in the past. If this is the case a system is characterised as *path-dependent*. Theoretical, historical, and empirical studies have argued about the effects of path dependence in many fields including the formation of government policies (Hacker, 2002), the choice of technologies (Nelson & Winter, 1982; David, 1985; Arthur, 1994), to the location of cities (Arthur, 1994; Page, 1998) and organisational development (Sydow et al., 2009). Page (2006) characterises four general mechanisms that may lead to path dependent development in a system: increasing returns, self-reinforcement, positive feedbacks, and lock-in. Path-dependence applies to many CAS and can often help explain differences in the development of systems that appeared to be similar at a previous point in time.

Through interaction and feedback effects CAS can exhibit sudden changes in their aggregate behaviour, triggered even by minute changes in the system. First observations of such an effect once gave rise to the development of Chaos Theory - the study of low dimensional, dynamic systems, which have been used to model market dynamics and competition (Hibbert & Wilkinson, 1994). It may not be possible to precisely predict the state of a chaotic system, but it is possible to derive a set of probabilities to find the system in any given state at any point in time. These stochastic descriptions of dynamic systems are referred to as *attractor states*. A chaotic system may have numerous attractor states, and the initial conditions of the system will determine which of these attractors will be approached.

This approach can serve as a template for the analysis of CAS. CAS also are dynamic systems, but typically they have a much higher dimensionality.Yet, their analysis follows essentially the same principles: First, identify and describe the states that a system can reach, focussing especially on its emergent patterns and regularities. Second, relate these states to the initial conditions that bring them about, and systematically assess the probabilities for each of these states, conditional on initial conditions.

Muliple interdependencies and feedback effects can lead to very different outcomes even when the system is initiated under very similar conditions. Therefore it is often crucial to assess the *robustness* of outcomes of a complex system. Computer models that repeat near identical simulation runs numerous times can be a very helpful tool to identify the distribution of possible developments.

Following a similar line of thought the patterns that emerge in a CAS can be assessed regarding their *resilience*. This refers to a system's behaviour with regard to the impact of external shocks, i.e. does the systemic behaviour change in response to external forces or is it able to absorb these? In this way, likely *tipping-points* can be identified that, once reached, change the aggregate patterns in the system, essentially moving it to another attractor state. These transitions can happen abruptly and lead to fundamentally different behaviour (Fisher, 2011).

Understanding complex systems requires suitable tools, among them ABMs. In the next section we will turn to practical considerations regarding the realisation of an ABM project that may help structure research in this area and better anticipate its peculiarities.

3. The Process of Developing ABMs

The phenomena that we may seek to better understand with the aid of ABM are aggregate patterns of behaviour that arise through interactions of interdependent individuals. Examples of these include diffusion processes for information or product adaptation (Goldenberg et al., 2002; Delre et al., 2007; Watts & Dodds, 2007; Delre et al., 2010), competition in an industry (Midgley et al., 1997; Midgley & Marks, 2004; Følgesvold

& Prenkert, 2009) or the self-organisation of a business network (Held, 2013). Computational models can also represent upgrades and refinements of existing theories that explain aggregate patterns through the activities and interactions of individuals, for example explaining how market-level diffusion models can be motivated by individuals' decisions (Goldenberg et al., 2001), or how market-clearing prices are brought about by pairwise negotiations (Axtell, 2005). The range of possible applications for ABM is large, as it is suited to shed a new light on existing models and help identifying new complex processes alike.

Due to the nature of the method, the development process of ABMs differs from classical hypothesis development and testing. Cioffi-Revilla (2010) argue that complex systems models need to reach a sufficient level of detail to be able to complete empirical tests successfully. Only if we create an artificial world that is sufficiently realistic can we expect to observe realistic patterns in it. It is part of the model development process to identify a sufficient set of assumptions and their implementations that brings about the desired aggregate patterns. Often this process is iterative, incorporating new insights from the implementation in the theory and vice versa. In the following we will present suggestions of how to deal with these new challenges and illustrate them with examples from the development process of ABMs concerning the self-organisation of a business network. The models discussed have been implemented in NetLogo (Wilensky, 1999) and are available for download under http://snurl.com/273mhyj. They were developed consecutively and reflect the iterative learning process that will be discussed in Sec. 3.3.

3.1. Identification of Mechanisms

Social mechanisms describe how the actions and interactions that constellations of individuals engage in bring about social phenomena. Therefore, a theory explaining such a phenomenon needs to identify the relevant individuals with their relevant characteristics, and explicate the activities that they engage in, and how they affect each other. Marketing and related literature that focusses on the behaviour and decision making can serve as a guideline to outline the potential candidates. Mechanisms and processes may be identified based on theory or through the observation of real systems over time (Bairstow & Young, 2012; Huang & Wilkinson, 2013). They may be implemented in various ways in an ABM. It may be helpful to group them, as there may be more specific and more general types of mechanisms relevant to the problem at hand. Moreover, a concise summary of the potential candidates may help decide which mechanisms are essential to a simplified baseline model, and which ones can be left out or maybe added later in the process.

Considering the example of mechanisms in the development of business relationships and networks, Tab 1 classifies and gives examples of the different types of mechanisms involved in establishing and developing relations between people and firms in business markets.

Further to the mechanisms summarised in Tab. 1 models would need to take into account how these activities are affected by the prevailing environmental conditions, including the natural, social and cultural environment.

The exemplary models contain various combinations of these mechanisms as illustrated in Tab. 2. Business acting and specialising are considered paramount to all other mechanisms as they constitute the underlying practical and economic rationale for cooperation and coordination. Therefore they are contained in each of the models. However, the individual models place different emphasis on the remaining classes of mechanisms. Concretely, ABM1 simulates specialisation and exchange on a fixed, random and exogenous network structure, while ABM2 endogenises the process of network formation, including mechanisms of searching, comparing and terminating. ABM3 returns to a fixed structure of interactions, however, it now allows the researcher to control the structure and investigate its effects.

3.2. Implementation of Modules

It is good practice to implement the actual code of an agent-based model in a modular fashion. This means that the individual rules that govern the agents' decision-making, as well as their actions and interactions, are coded independently and separately from each other. Each mechanism should be conceptualised as a separate module of code, although it is possible that some machanisms are only represented implicitly in the implementation of others. This is a standard practice in software design, ensuring that individual modules can be substituted with alternative implementations without affecting the rest of the simulation.

General	Specific Mechanisms
Business Acting and Specialising	Specialising (Smith, 1776), Learning (Wright, 1936; Yelle, 1979), Increasing scale (Florence, 1933; Dixon & Wilkinson, 1986), Combining (Richardson, 1972; Baldwin & Clark, 2000), Limiting (Stigler, 1951; Shove, 1930), Growing (Boulding, 1953; Penrose, 1959), Intermediating (Hall, 1949), Outsourcing (Robinson, 1931), Coordinating (Commons, 1931; Coase, 1937; Richardson, 1972), Exploration and Exploitation (March, 1991)
Business Mating	Finding, Being found, Searching (Frazier, 1983), Defining criteria (Anderson & Narus, 1991), Informing (Havila & Wilkinson, 2002; Li & Rowley, 2002), Promoting (Dwyer et al., 1987), Targeting (Hedaa, 1996), Assortative mating (Wilkinson et al., 2005), Evaluating (Anderson & Narus, 1991)
Business Dancing	Getting Acquainted (Dwyer et al., 1987), Learning (Narayandas & Rangan, 2004), Negotiating (Bergen et al., 1992; Mallen, 1967), Bonding (Bitner, 1995; Grönroos, 1994; Narayandas & Rangan, 2004), Sensemaking (Welch & Wilkinson, 2002), Socialising (Håkansson, 1982; Heide & John, 1990), Coordinating through contracts (Grossman & Hart, 1980; Hart & Moore, 1990), Coordinating through other arrangements (Osborn & Baughn, 1990; Rindfleisch & Heide, 1997; Palay, 1984; Noordewier et al., 1990), Coordinating through power (Wilkinson, 2008), Coordinating socially (Wilkinson & Young, 1994; Huang & Wilkinson, 2013), Monitoring (Eisenhardt, 1989), Adapting (Williamson, 1975, 1983, 1996), Cheating (Williamson, 1975, 1985, 1996), Terminating (Anderson et al., 1994; Young & Denize, 1995; Halinen & Tähtinen, 2002)
Interconnecting Relations	Prioritising (Turnbull et al., 1996), Comparing, (Anderson & Narus, 1984), Intermediating (Håkansson, 1982; Burt, 1992, 2004), Competing (Balderston, 1958), Communicating (Håkansson & Johanson, 1988; Havila & Wilkinson, 2002), Transmitting (Easton & Lundgren, 1992; Blankenburg-Holm et al., 1996; Wiley et al., 2009), Clustering (Marshall, 1898, 1919)

Table 1: Summary of the social mechanisms that drive the development of business relation and networks

Mechanism	ABM1	ABM2	ABM3
Specialising	✓	✓	✓
Increasing scale	✓	✓	✓
Searching		✓	
Coordinating through other arrangements	✓		
Coordinating through contracts		✓	✓
Negotiating	✓	✓	✓
Learning	✓	✓	✓
Adapting		✓	✓
Terminating		✓	
Prioritising	✓		
Comparing		✓	✓
Intermediating		✓	✓

Table 2: Summary of the modules implemented in ABM1-3. Note that implementations of each mechanism may differ between models.

Based on this modularity of mechanisms, it can be a useful strategy to start from a very simple model, which is easy to specify, implement and understand. This baseline model can then be extended to encompass more features and more complexity. Gilbert (2004) draws an analogy to hypothesis testing: Modules can be substituted and added to the baseline model and changes in the aggregate outcomes can then be attributed causally to the changes in the set of included mechanisms.

Further to these theoretical considerations, modular coding substantially simplifies the verification process of the code. Verification is the process of ensuring that the code does what it should do, according to the model specification - the underlying theory. This includes the identification of coding errors, but also checking for logical errors and deviations between spoken-language theory and computer-language code (Midgley et al., 2007; Railsback & Grimm, 2011). Software modules should be scrutinised regarding their internal consistency, their congruence with the theory and their response to extreme as well as unexpected values. Other means of verification include defensive programming, which tries to anticipate potential misusage of a module and includes warning messages and other precautions in the code, parallel coding of the same code by another programmer, and peer review.

Here we will highlight only one such module: Specialisation. ABM1 and ABM2 use the same implementation, a variation of a logistic function that takes as its input the agent's prior production experiences $V_i = (v_{i1}, \ldots, v_{im}), v_{ij} \in \mathbb{R}$ and maps this vector to a vector $R_i = (r_{i1}, \ldots, r_{im}), r_{ij} \in [0.5\ 1.5]$ that represents the productive capabilities of that agent. This specialisation function is

$$r_{ij} = 0.5 + \frac{1}{1 + \exp^{-\frac{v_{ij} - \mu(V_i)}{\sigma(V_i)}}}, \tag{1}$$

where $\mu(V_i)$ is agent i's mean production experience across all products so far, and $\sigma(V_i)$ is the standard deviation. The calculations go as follows: First the agent's production experience is normalised across all goods, so that the distribution has mean 0 and a standard deviation of 1. Second, the normalised values are mapped onto a S-shaped function bounded by 0 and 1, and finally these values are shifted up by 0.5 to yield coefficients for the agent's production function that maps input (time/effort) to outputs (products). The algorithm is illustrated in Fig. 1.

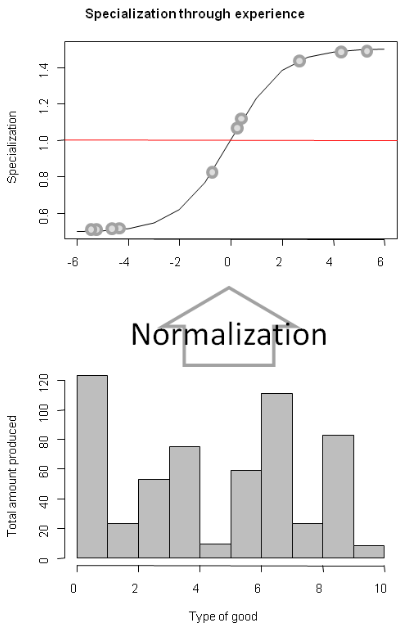

Figure 1: Schematic illustration of the specialisation algorithm that maps the agent's production experience to its production capabilities, coefficients for the production function in the range of 0.5 to 1.5.

The idea behind this algorithm is twofold: on one hand it models that agents improve what they are doing by doing it - a basic form of behavioural learning. On the other hand, the algorithm takes into account that the theoretical limit for all production experience is positive infinity. Given enough time, every agent's experience in the production of any good will likely surpass any fixed threshold. Therefore, the algorithm normalises experience values, so that productive capabilities for one type of good are relative to the agent's experience across all goods.

3.3. The Modelling Cycle

As indicated above, the development of a theory about emergent phenomena and the implementation of an ABM to simulate them are interdependent processes that feed back into each other. Complex systems can be counter-intuitive and ABM is a tool to assist developing that intuition. This means that it is to be expected that the theory and model will have to be revised during the process and on the way to a satisfying model

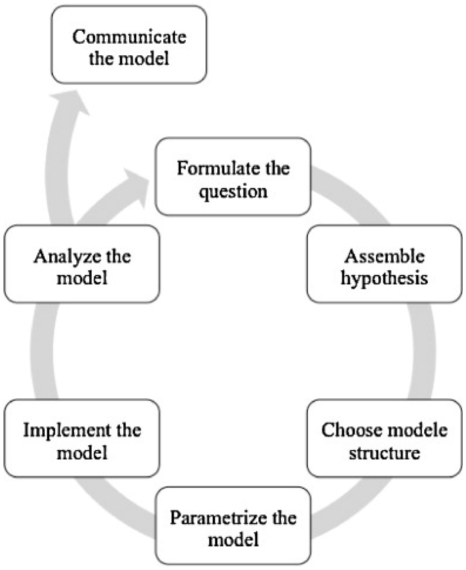

Figure 2: The six step modelling cycle, according to Latombe et al. (2011)

valuable insights can be gained from understanding why unsuccessful implementations failed to produce the phenomena under consideration.

Describing this iterative process Grimm et al. (2005) coined the term *Modelling Cycle*. An illustration updated by Latombe et al. (2011) is reproduced in Fig. 2. It summarises six tasks that should be performed cyclicly until the model exhibits the desired aggregate patterns: (a) formulate the question, (b) construct the theory as a set of mechanisms (hypotheses), (c) choose the model structure, (d) identify empirical data to be included in the model (parameterisation), (e) implement the model, (f) analyse the model. The Modelling Cycle emphasises that this is to be seen as an iterative process, and revisions can affect any of the stages in the process. Once the model gives enough confidence in answering the original question, we can proceed with the seventh task, that is (g) communicate the model.

Table 2 illustrates such an iterative process of readjustment and refinement of models. The overarching quest guiding the research project in which ABM1-3 were developed was to better understand the dynamics and evolution of business relationships and networks. The overview of relevant mechanisms in Tab. 1 was used to develop different theories - and models - that illustrate different aspects of this phenomenon. ABM1 had a narrow focus on investigating the mechanics of specialisation of individuals in a group. It was followed by ABM2 which built on these insights and shifted attention towards the search and mating mechanisms driving the network's development. ABM3 then took a step back and used a different implementation of the specialisation algorithm and highlighted the effect of intermediation on exogenously determined network structures. In the course of this research project a total of seven different ABMs were developed (Held, 2013, discusses this process). Other models both preceded and followed the models mentioned here.

A key driver in the transition from ABM2 to ABM3 were concerns about the validity of the original implementation of the specialisation mechanism. It became apparent that the algorithm from Eq.1 resulted in an unintended form of "unlearning", where gaining experience in the performance of one task unavoidably leads to the decrease in productive capabilities for others. In response the specialisation mechanism in ABM3 was adapted to

$$r_{ij} = 0.5 + \frac{1}{1 + \exp^{-\frac{v_{ij} - \mu(V_i)}{\sigma(V_i)} * \log_{100} \sum V_i}}. \tag{2}$$

The upgraded specialisation function reduces the effect of "unlearning" substantially. It introduces a term that increases the range of possible levels of specialisation in parallel to the increase of overall experience. This satisfies three requirements at the same time: the more often a task is performed, the better an agent becomes at it, the learning curves are steep at the beginning of the process and flatten out towards the end, further improving on the capabilities to perform one task may reduce the capabilities of performing another, but the magnitude of the negative effect is below the magnitude of the positive effect.

A graphical representation of this updated specialisation function is provided in Fig. 3. It illustrates the shift of the potential for specialisation as the agent gains more production experience. Depending on $\log_{100} \sum V_i$, the function is either a short, nearly straight line between 0.98 to 1.02, or, with increasing experience, extending to the s-shape used in previous models (highlighted to the right of the image).

To better understand the effects of the new specialisation mechanism, the specialisation process of a single agent was monitored in a series of small-scale experiments. In

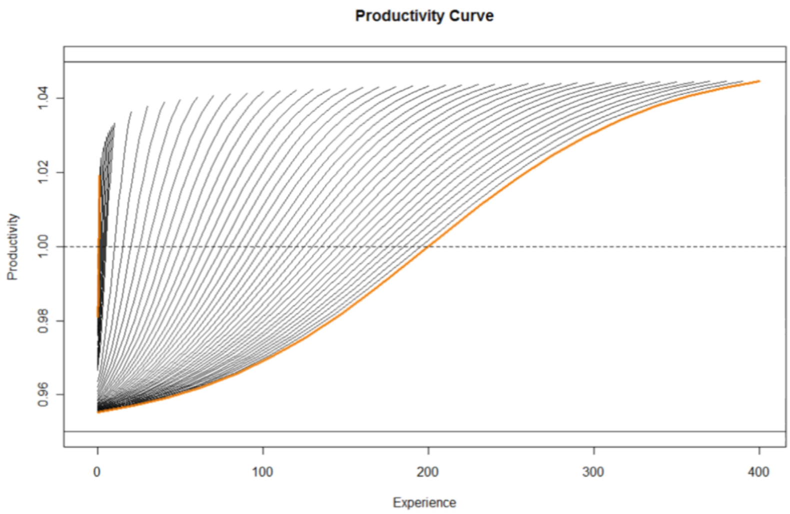

Figure 3: Graphical representation of the updated specialisation function in ABM3. The potential for specialisation shifts as the agent gains more production experience.

a system with only one agent and two goods, the agent will alternate between the production of either good. Initial small differences in production experience will eventually lead to specialisation, but on a comparatively low level. With three goods, the potential paths of specialisation for one isolated agent look different. About a third of the model runs lead to positive developments and increases in wealth, while the other two-thirds lead to stagnation and depletion of funds. The exact mechanics for these outcomes are revealed in Fig. 4. It compares the production slopes upon initialisation to those upon completion of 100 model runs. It becomes apparent that the initial conditions determine the course of the agent's specialisation and performance in the rest of the model. An agent will perform well and on average create gains through production only if two out of the three production slopes are relatively steep from the beginning on, while one is very flat by comparison. The production of two goods with steeper slopes will then over compensate the losses incurred in the production of the third. Like in the case with two goods, these differences will be maintained throughout the run because the individual's consumption function requires equal amounts of each good. Further developments of this model involve introducing mechanisms of searching and mating in order to model the development of exchange networks. This is beyond the scope of this article, which is an introduction to ABM methods. The models will be described in subsequent publications.

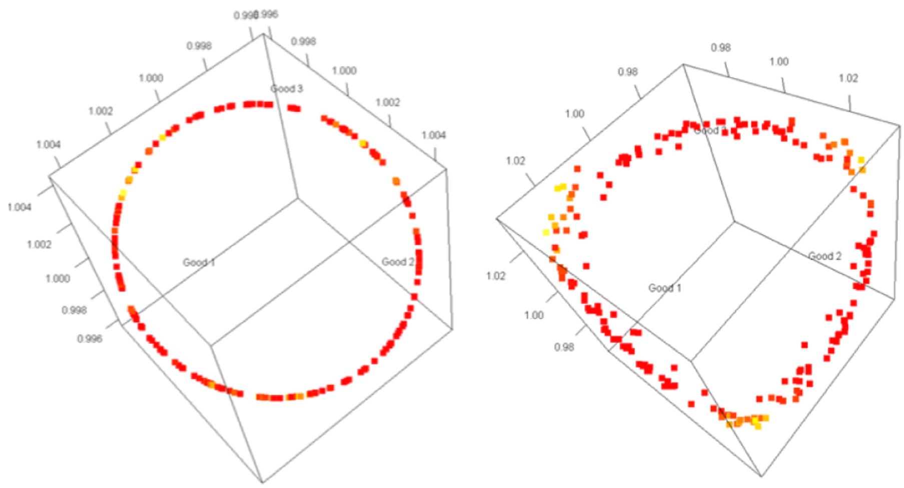

Figure 4: Agent specialisation and performance with one agent and three goods. Slopes of the agent's production functions individually for each good, upon initialisation (left) and after 100 rounds (right).

4. Discussion and Summary

This paper discussed selected aspects concerning the use of Agent-based Modelling for the development of theories about emergent social phenomena in marketing and the social sciences in general. It addressed both theoretical aspects about the types of phenomena that are suitably addressed with this approach and practical guidelines to help plan and structure the development of a theory about the causes of such a phenomenon in conjunction a matching ABM.

It needs to be emphasised that ABMs facilitate the formal representation and analysis of phenomena that cannot readily be addressed by traditional tools such as regression analysis or differential equations. ABM are designed to study the emergence of aggregate patterns that arise through the constellations of interdependent actions and interactions of numerous, possibly heterogeneous entities. The underlying assumptions are grounded in the paradigm of complex adaptive systems, that highlights the importance of adaptive behaviour and dynamic feedback between the individuals and the aggregate level of a system. ABMs are a helpful tool when interactions matter - when the whole is more than the sum of its parts.

Due to the comparative novelty of the ABM approach, this article began with a general discussion of what we understand under the term "model". The main point of its argument is that a model can be much more than just a statistical test for a single hypothesis. We use the term much more generally, characterising models as simplifications of the real world, which are less detailed and/or less complex than the original and therefore more easily accessible and understandable. Designed for a specific purpose, models seek to capture the essential aspects of their original template and allow us to do something with them that is not possible with the original or let us understand more easily the causal relations in the original by highlighting only the most essential aspects without distracting details. Models are just analogies, not the real thing, but understanding a model can help us better understand the original.

Such simplified and more easily accessible models are especially important when it comes to the understanding of complex adaptive social systems, that often are difficult to monitor and influence, as well as to intuit and theorise about. Consequently the construction of a ABMs serves two purposes simultaneously: theory testing, but also theory development. This may be unusual, even frowned upon, in more established forms of scientific inquiry, but seeing that this is largely about fundamental research concerning marketing and social systems this must be accepted as a necessity.

To help develop ABMs and matching theories, this paper presented two sets of guidelines, first theoretical and second practical in nature. The theoretical guidelines may serve as a lens to identify the right problems and the domain of possible answers: ABM can model the causal processes that bring about patterns in a population of interdependent entities. Such patterns include spatial distributions, rates of diffusion, typical actions, beliefs or desires, or informal rules, to name only a few. Following the principles of Analytic Sociology, causal explanations for such patterns should be sought in the domain of the individuals' actions, interactions, their properties and their constellations - not in the co-variations of abstract variables!

The practical guidelines presented here aim to help anticipate and structure the research process that may require the simultaneous co-development of both theory and model. The main point here is that a cyclical development process is to be expected, and should not be seen as a disproof of the entire project in the early stages. The theories/models of complex systems need a certain level of detail and complexity to bring about the patterns under examination. Therefore they necessarily have to combine a hard core of theoretical assumptions and a collection of auxiliary hypotheses, and each of them, or any combination of them, could potentially be the reason why a certain pattern does not emerge as expected. Investigating these effects is a necessary part of both model construction and theory development.

In order to better manage this cyclical learning process, this paper recommended two practical solutions: The compilation of candidate mechanisms from the existing literature and the implementation of these mechanisms in a modular fashion. Marketing and related literature has long studied the behaviour and decision making of individuals, although it rarely considered as social mechanisms. But such mechanisms are often implied and a compilation of existing mechanisms can help identify and prioritise them for the process of model/theory development. Regarding the practicalities of their implementation in computer code, it may be helpful to conceptualise them as individual software modules, so that they can be changed and varied more easily and in a systematic, experimental fashion.

The investigation of marketing and social phenomena from a perspective of emergent patterns in complex systems is still a comparatively novel approach and in marketing this kind of thinking is still new, but successful demonstrations of the potential of ABM have already been published. The main goal of this paper was to make this kind of research more accessible and help anticipate and structure the research process it may require. It is different, it is new, but in a constantly more interdependent world it may be necessary to embrace this kind of thinking to better understand and sustain the world we live in (Helbing, 2013).

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