LECTURE 4: ELASTICITY

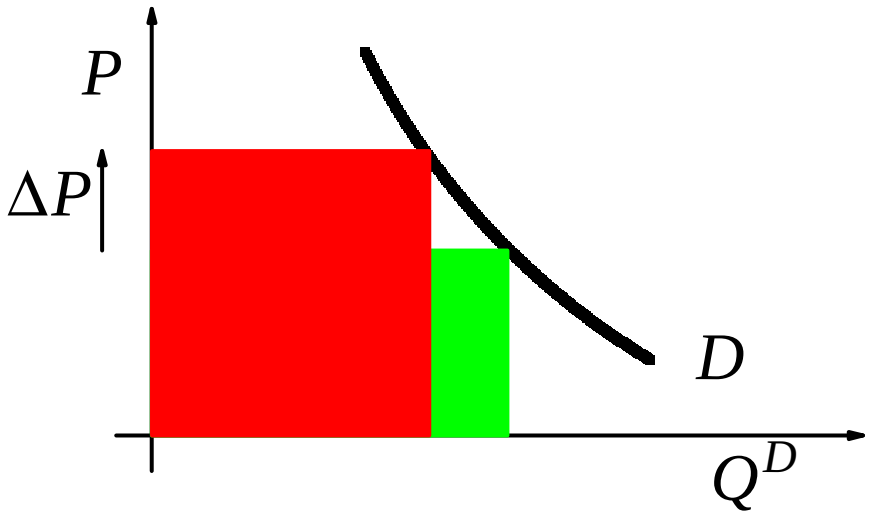
Today's Topics

1. **The Price Elasticity of Demand: total revenue, determinants, formulæ, a bestiary, total revenue, estimation of price elasticity of demand.**
2. **The Income Elasticity of Demand, and the Cross-Price Elasticity of Demand.**
3. **The Elasticity of Supply: determinants, formula.**
4. **Two Applications: the OPEC cartel tries to keep the price of oil up, farmers' adoptions lower their profits.**

REVENUE AND PRICE

Managers make decisions at the margin — a little more, a little less — and only have reasonable information about a small region on the demand curves they face.

Q: How does Revenue (Price $\times$ Quantity) change when we raise the price we sell at?



ALGEBRAIC DERIVATION

A: It depends on how much the quantity demanded falls as we move up the demand curve to $P+\Delta P, Q+\Delta Q$.

Old Revenue: $R = P \bullet Q$

New Revenue: $R' = (P + \Delta P) \bullet (Q + \Delta Q)$

$$= P \bullet Q + P \bullet \Delta Q + Q \bullet \Delta P + \Delta P \bullet \Delta Q$$

Ignoring $\Delta P \bullet \Delta Q$**, the change in revenue is:**

$$R' - R = P \bullet \Delta Q + Q \bullet \Delta P$$

$$= Q \bullet \Delta P \bullet \left(\frac{P}{Q} \frac{\Delta Q}{\Delta P} + 1 \right) = Q \bullet \Delta P \bullet (\eta + 1),$$

Is $\eta = \frac{P}{Q} \frac{\Delta Q}{\Delta P}$ **greater than, equal to, or less than −1?**

INTUITION OF THE REVENUE CHANGE

$\eta \equiv \dfrac{\Delta Q/Q}{\Delta P/P}$ **is the *price elasticity of demand.***

That is: if the percentage fall in quantity demanded $\Delta Q/Q$ is greater than the percentage rise in price charged $\Delta P/P$, then the Revenue will fall.

So:

$\eta < -1$	**elastic**	$\lvert\eta\rvert > 1$	$\leftrightarrow$	$\Delta R < 0$
$\eta > -1$	**inelastic**	$\lvert\eta\rvert < 1$	$\leftrightarrow$	$\Delta R > 0$
$\eta = -1$	**unitary**	$\lvert\eta\rvert = 1$	$\leftrightarrow$	$\Delta R = 0$

$\therefore$ **Taxes on what?**

To summarize:

	$\lvert\eta\rvert$	Price	Total Expenditure (Revenue)
Elastic demand	**> 1**	**Up**	**Down**
		Down	**Up**
Unitary elasticity	**= 1**	**Up**	**Constant**
		Down	**Constant**
Inelastic demand	**< 1**	**Up**	**Up**
		Down	**Down**

Price Elasticity of Demand and Revenue Changes

PRICE ELASTICITY OF DEMAND

Elasticity **is a dimensionless measure of the sensitivity of one variable to changes in another, cet. par.**

The *price elasticity of demand* η is the percentage change in quantity demanded Q divided by the percentage change in the price, P.

Because of The Law of Demand, if ΔP is positive (price rises), then ΔQ cannot be positive, and in general is negative.

Since the price elasticity of demand is never positive, we usually ignore its sign (or use its absolute value $|\eta|$).

FOUR DETERMINANTS OF η

Four determinants of a good's *own-price elasticity of demand* η:

1. **Necessities v. discretionary goods (or luxuries): necessities tend to have inelastic demands; luxuries have elastic demand. Depends on the buyer's preferences. Examples?**
2. **Availability of close substitutes: the greater the number of available substitutes, the more elastic the demand. Examples?**

DETERMINANTS

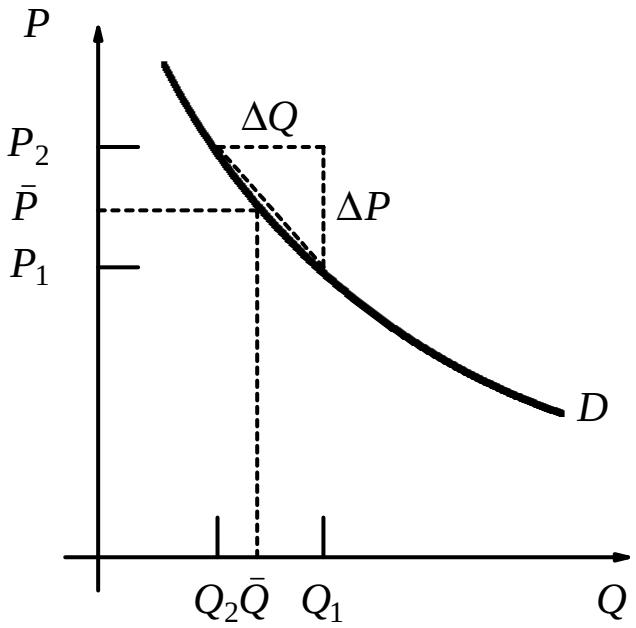
3. **Definition of the market: broad definitions (e.g. food) have less elastic demands than do narrowly defined markets (e.g. Nestlé's chocolate) which have more substitutes. Examples?**
4. **Time horizon: the greater the time horizon, the easier for consumers to find substitutes, or make do without, so the more elastic the demand.**
   Examples?

(These properties do not follow from the axioms and definitions; they have been observed in the market.)

ARC OR POINT MEASUREMENTS

The arc elasticity: $\eta = \dfrac{\Delta Q / \bar{Q}}{\Delta P / \bar{P}} = \dfrac{\bar{P}}{\bar{Q}} \dfrac{\Delta Q}{\Delta P} \leq 0$

using mid-points: $\bar{P} \equiv \frac{1}{2}(P_1 + P_2), \bar{Q} \equiv \frac{1}{2}(Q_1 + Q_2)$



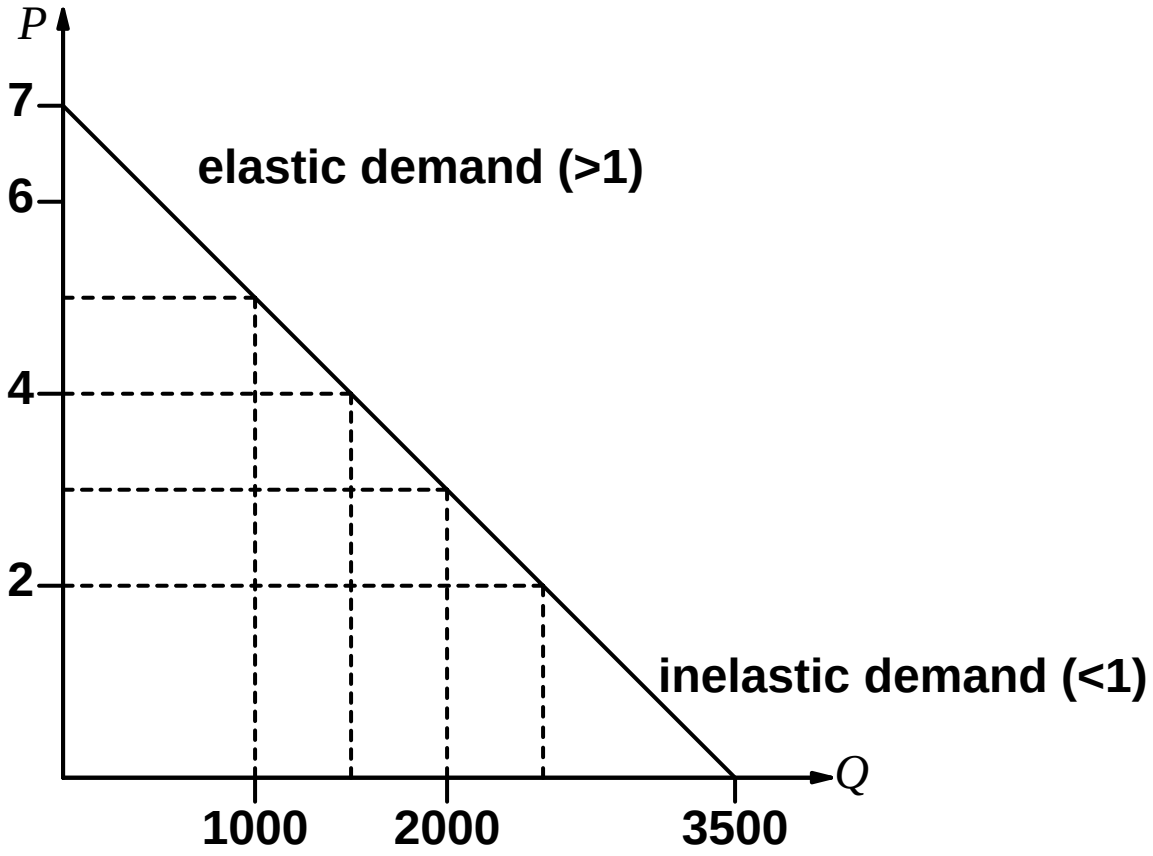
A LINEAR DEMAND SCHEDULE

Elasticity is not equal to the slope of the demand curve. **Indeed, we can calculate the price elasticities along a linear demand curve. (*Arc elasticities*, midpoint convention.)**

Price ($/t)	Purchase (tonnes)	Value of Sales ($)	$\|\eta\|$ Elasticity
2	2500	5000	5/9 = 0.556
3	2000	6000	1
4	1500	6000	9/5 = 1.8
5	1000	5000	

$$\text{eg. } \frac{5}{9} = \frac{(2,500 - 2,000)/2,250}{(3-2)/2.5} = \frac{\Delta Q/\bar{Q}}{\Delta P/\bar{P}}$$

A LINEAR DEMAND CURVE

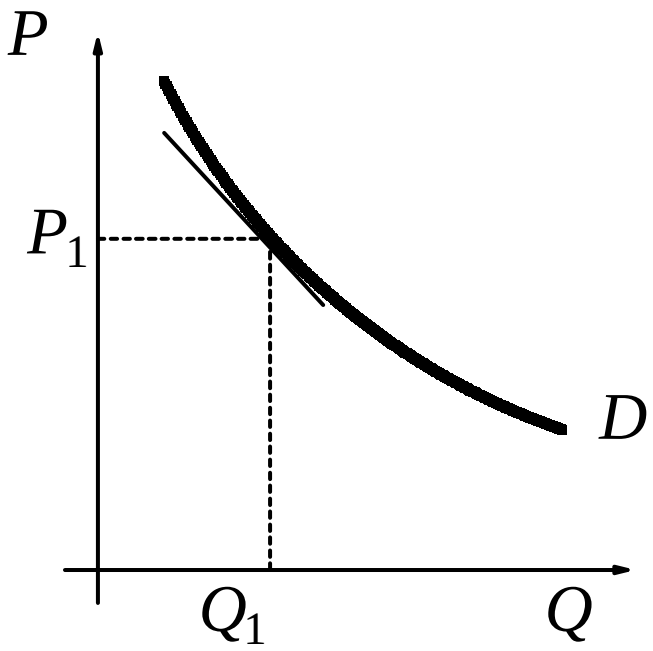


$Q^D = 3500 - 500P$

elasticity ≠ slope

POINT ELASTICITY

The point elasticity: $\eta = \frac{P_1}{Q_1}\,\frac{\partial Q}{\partial P}$**, where** $\frac{\partial P}{\partial Q}$ **is the slope of the curve at the point** P_1, Q_1**. (The partial derivatives** ∂ **imply ceteris paribus: only price** P **is changing.)**

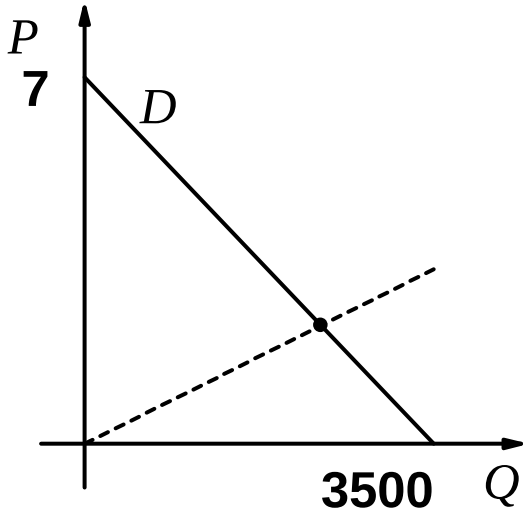


POINT ELASTICITY FORMULA

A linear demand function: $Q = 3500 - 500P$**, or** $P = 7 - Q/500$**. The slope is** $\partial P/\partial Q = -1/500$.

$$\therefore \quad \eta = -500\, P/Q$$

NB: elasticity varies along a straight line.



$$\text{Elasticity at point} = \frac{\text{the slope of the ray through the origin}}{\text{the slope of the demand curve}}$$

A BESTIARY OF DEMAND CURVES

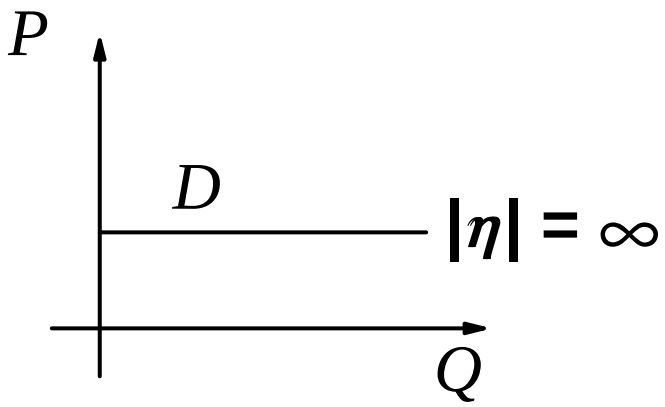
When $|\eta|$ > 1 we have *elastic* demand

= 1 we have *unitary elastic* demand

< 1 we have *inelastic* demand

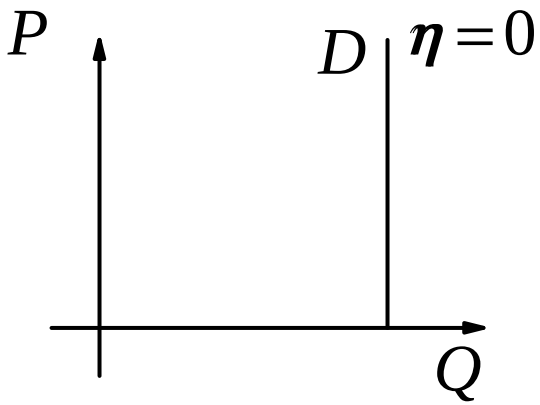
= 0 we have *perfectly inelastic* demand

$\rightarrow \infty$ *perfectly elastic* demand

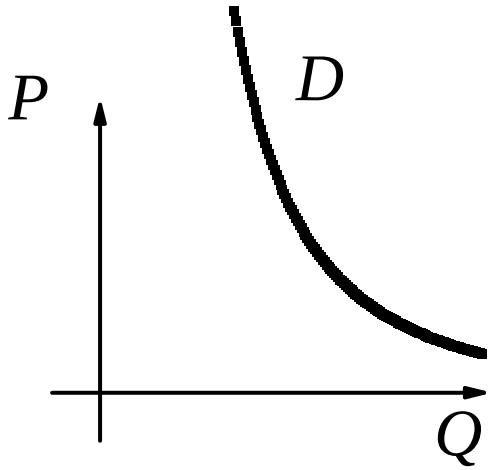


Horizontal demand: perfectly elastic.

A BESTIARY 2

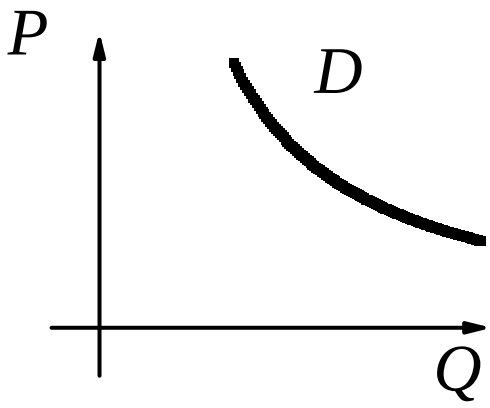


Vertical demand: perfectly price-inelastic.



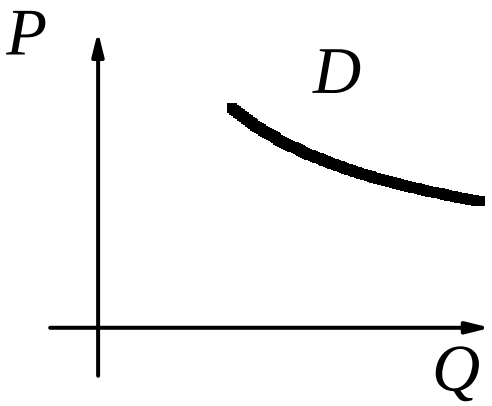
|η| = ½: inelastic demand

A BESTIARY 3



$|\eta| = 1$ unitary

(A rectangular hyperbola.)



$|\eta| = 2$: elastic demand

INCOME ELASTICITY OF DEMAND ε

The proportional *change in the amount demanded* in response to a 1 percent *change in income* I.

Or algebraically:

(arc) $$\varepsilon \equiv \frac{\Delta Q^D / Q^D}{\Delta I / I}$$

(point) $$\varepsilon \equiv \frac{\partial Q^D}{\partial I} \frac{I}{Q^D}$$

***Normal* goods have positive income elasticities.**
***Inferior* goods have negative ε.**
***Luxuries* have $\varepsilon > 1$.**
***Necessities* have $\varepsilon < 1$, but positive.**

Examples?

CROSS-PRICE ELASTICITY OF DEMAND

The percentage change in the demand X^D for good X in response to a 1 percent change in the price P_Y of good Y.

Arc measure:

$$\eta_{X,Y} \equiv \frac{\Delta X^D / \bar{X}^D}{\Delta P_Y / \bar{P}_Y}$$

Point measure:

$$\eta_{X,Y} \equiv \frac{\partial X^D}{\partial P_Y} \frac{\bar{P}_Y}{\bar{X}^D},$$

where $\Delta X^D = X_1^D - X_2^D$, $\Delta P_Y = P_{Y1} - P_{Y2}$,

using midpoints: $\bar{X}^D = \frac{1}{2}(X_1^D + X_2^D)$, **and** $\bar{P}_Y = \frac{1}{2}(P_{Y1} + P_{Y2})$.

SUBSTITUTES AND COMPLEMENTS

If $\eta_{X,Y}$ > 0 then X and Y are *substitutes*

< 0 then X and Y are *complements*

= 0 then X and Y are *unrelated*

Examples?

of substitutes?
of complements?

Note: in general $\eta_{X,Y} \neq \eta_{Y,X}$ (see Coke and Pepsi below) because of *income effects* (GKSM p.472).

ESTIMATING ELASTICITY

A constant-elasticity demand function can be written as

$$Q = A \bullet P^{\eta}$$

where η is the price elasticity of demand, and A is a constant.

Taking logarithms:

$$\log Q = \log A + \eta \log P,$$

or

$$y = a + \eta x$$

which means that we can use linear regression to estimate the elasticity η (assuming our data come from an unshifting demand curve).

MARKET DATA

Price, Cross-Price, and Income Elasticities of Demand for Coca-Cola and Pepsi

Elasticity	Coca-Cola	Pepsi
Own Price elasticity η	−1.47	−1.55
Cross-price elasticity $\eta_{X,Y}$	0.52	0.64
Income elasticity ε	0.58	1.38

Source: Besanko & Braeutigam, *Microeconomics*.

So a 1% increase in Coke's price led to a 1.47% fall in Coke's quantity sold and a 0.64% increase in Pepsi's sales.

(Perhaps estimated using $X^D = A \bullet P_X^{\eta} \bullet I^{\varepsilon} \bullet P_Y^{\eta_{X,Y}}$).

PRICE ELASTICITY OF SUPPLY

(Only for price-taking suppliers — monopolists do not have supply curves.)

The price elasticity of supply κ is the percentage change in quantity supplied Q^S per percentage change in price P:

$$\kappa = \frac{\Delta Q^S / Q^S}{\Delta P / P}$$

Can be perfectly inelastic (κ = 0, vertical), perfectly elastic ($\kappa = \infty$, horizontal), inelastic (κ < 1), and elastic (κ > 1).

Depends mainly on the time horizon: the longer, the more elastic, in general, because firms have more time to adjust their production processes in order to increase their profits.

OPEC AND THE OIL PRICE

In the early 1980s the OPEC cartel squeezed supply and pushed up the world price of oil. What happened?

In the short run, both supply and demand for oil are relatively inelastic:

- **changing capacity and proving up more reserves is relatively slow;**
- **old guzzlers and old habits of use are slow to change: demand adjusts only slowly.**

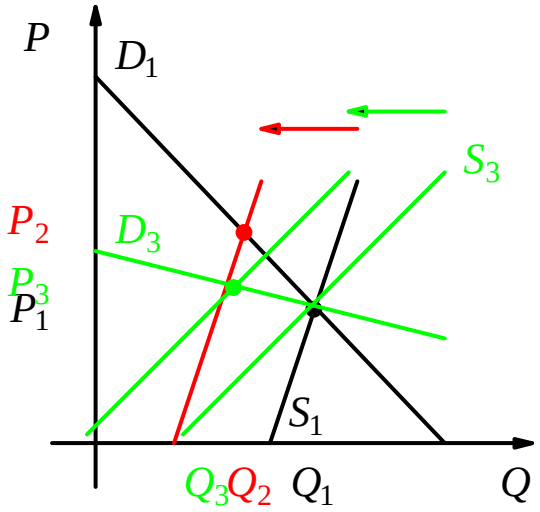
LONG-RUN MARKET ADJUSTMENT

In the long run, the higher price affected both supply and demand:

- **there was increased exploration → increased production, especially in the non-OPEC oil producers, such as?**
- **New R&D → more fuel efficient vehicles and industrial processes and household machinery, and these were eventually bought and installed to cut fuel bills → lower demand (than otherwise).**

The initial high price fell, although only slowly, and not (at first) back to the pre-squeeze price.

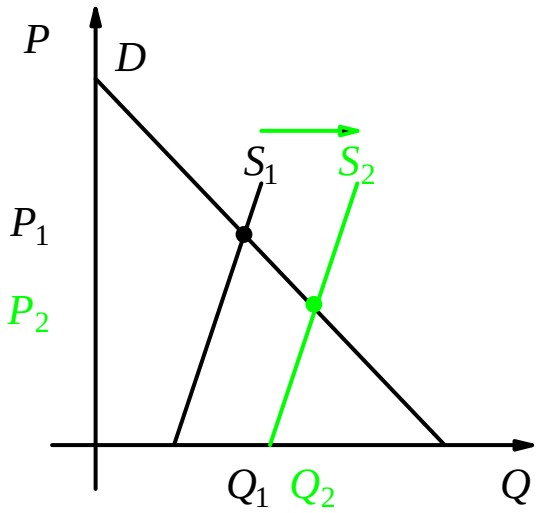
GRAPHICALLY



Over time, both supply and demand become more elastic: the later price P_3 is lower than the earlier price P_2, and the later quantity Q_3 is lower than the earlier quantity Q_2. OPEC cannot long sustain the high price P_2.

ARE FARMERS IRRATIONAL?

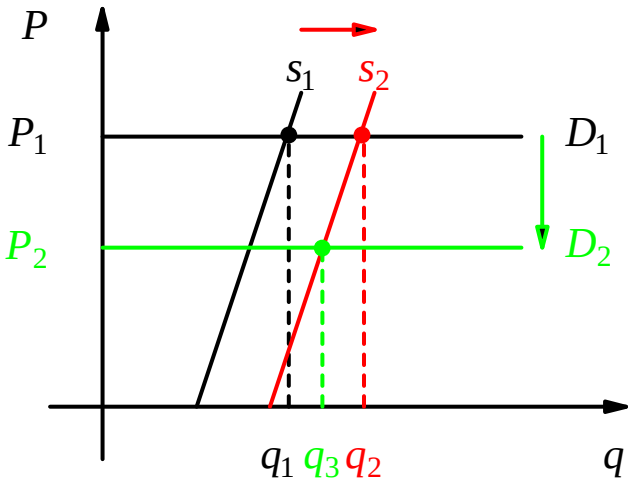
Farmers adopt new technology which reduces their costs. But such technology, when all adopt it and market supply expands, lowers their output prices. Why do they adopt it?



The industry view: downwards-sloping demand. With inelastic demand, revenues fall with price.

THE PRICE-TAKING FARMER

From the small (price-taking) farmer's view, the market price is a given: she faces an infinitely elastic (horizontal) demand curve, the going price. She adopts the new technology to improve her net returns or profits, by reducing her costs. Her supply curve expands.



As all farmers adopt the technology, price will fall. No single farmer, however, can prevent this.