

Strategic Interaction

Guess Two-Thirds of the Average

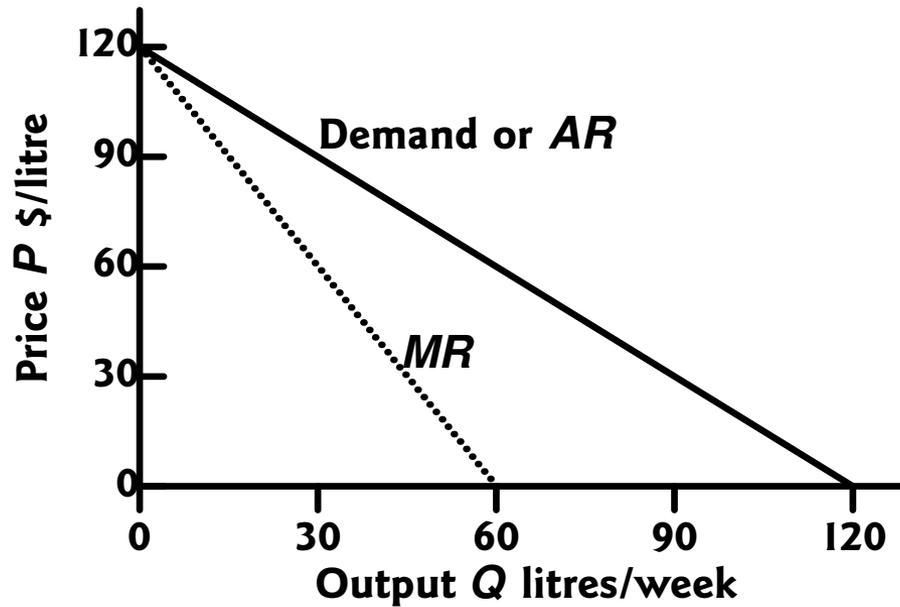
- Choose a number between 0 and 100.
- A prize of \$10 will be split equally between all participants whose number is closest to $\frac{2}{3}$ of the average number chosen (the mean of all choices).
- What should you choose?
- Write down your answer.
- If we repeated this several times, where would it end (its equilibrium)?

Today's Topics: Oligopoly

1. **Two Sellers:** price takers versus a monopoly (cartel) versus ...
2. **A Cournot Duopoly:** (pp. 322–28) payoff matrices, dominant strategies, Nash Equilibrium.
3. **The Prisoner's Dilemma:** (pp. 329–36) n -person games, the advertising game, repeated interactions.
4. **Other Games:** Chicken!, the macroeconomics game.
5. **Sequential Games:** game trees.

I. Two Sellers

Sellers Jack and Jill face this market:



The market demand curve.

In tabular form ...

Quantity (litres/week) <i>Q</i>	Price (\$/litre) <i>P</i>	Total Revenue <i>TR</i>	Marginal Revenue <i>MR</i> (\$/l)	Price Elasticity (arc)	$ \eta $ (equation)
0	120	0			∞
10	110	1100	110	23.0	11.0
20	100	2000	90	7.0	5.0
30	90	2700	70	3.8	3.0
40	80	3200	50	2.4	2.0
50	70	3500	30	1.67	1.4
60	60	3600	10	1.18	1.0
70	50	3500	-10	0.85	0.71
80	40	3200	-30	0.6	0.5
90	30	2700	-50	0.412	0.333
100	20	2000	-70	0.263	0.2
110	10	1100	-90	0.143	0.091
120	0	0	-110	0.043	0

Note: total revenue TR is a maximum when marginal revenue $MR = 0$;

for arc: $\eta = \frac{\Delta Q}{\Delta P} \frac{\bar{P}}{\bar{Q}}$, where \bar{P} and \bar{Q} are the midpoint measures;

for equation: $\eta = \frac{dQ}{dP} \frac{P}{Q}$

More or Less

Assume that marginal cost $MC = 0$ for all firm output y , for convenience.

Competition (price-taking):

choose output y^C to set Price $P^C = MC = 0$

$$y^C: MC(y^C) = 0 = P^C$$

$$\therefore Q^C = \sum y^C = 120 \text{ litres/week}, \pi^C = 0 \times 120 = 0.$$

Monopoly (Cartel):

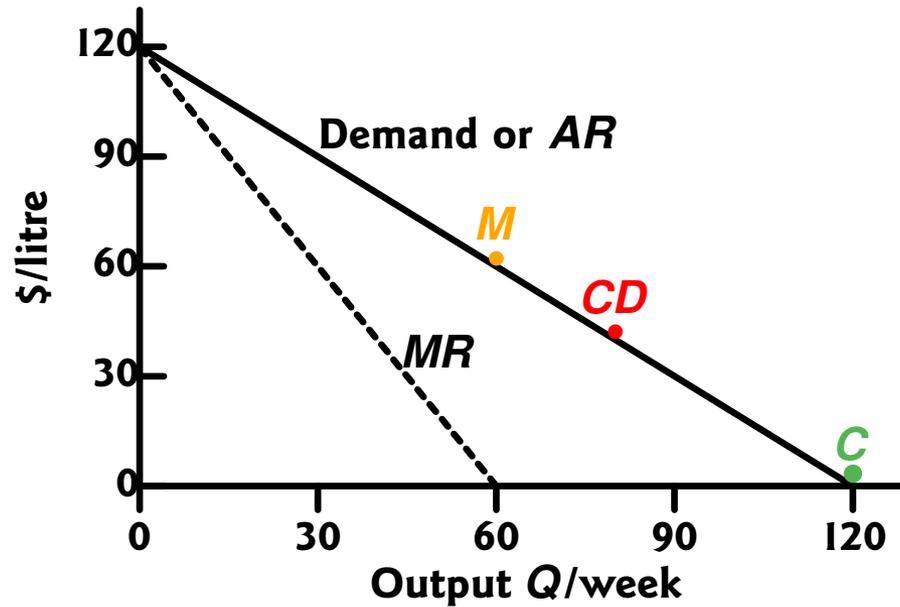
choose output y^M to set $MR = MC = 0$.

$$y^M: MR(y^M) = MC(y^M) = 0$$

$$\therefore Q^M = y^M = 60 \text{ litres/week}, P^M = \$60/\text{litre},$$

and $\pi^M = 60 \times \$60 = \$3600/\text{week}$

Graphically



Competitive: $P^C = \$0$, $Q^C = 120$.

Monopoly (Cartel): $P^M = \$60$, $Q^M = 60$.

Cournot duopoly: $P^{CD} = \$40$, $Q^{CD} = 80$.

A Cartel

What if J & J get together and agree on either the quantity to sell or the price at which to sell it? →

Collusion.

A group of sellers (or buyers) acting together forms a Cartel.

The two would act as a monopolist: selling 60 litres at \$60/litre.

How to split production and profits between them?

If equally, then each produces 30 litres and makes \$1800/week.

2. A Cournot Duopoly

If Jack assumes that Jill will produce 30 litres, what might he do?

- **Produce 30 litres and make \$1800/week, or**
- **Produce 40 litres and make ... what?**

$$Q = 30 + 40 = 70 \text{ litres} \rightarrow P = \$50/\text{litre.}$$

$$\text{Jack's profit} = 40 \times \$50 = \$2000 > \$1800/\text{week.}$$

Looks good.

**At 30 litres, Jill's profit falls to $30 \times 50 =$
\$1500/week.**

But if Jill thinks like Jack, then she also produces 40 litres, and $Q = 40 + 40 = 80 \rightarrow P = \40 , and the profit of each = \$1600/week.

Payoff Matrix I

Each player has two actions to choose from: produce 30 litres or produce 40 litres.

Their decisions are made independently: model with a 2×2 matrix, where Jack chooses which Row (top or bottom) and Jill chooses which Column (left or right).

		<i>Jill</i>	
		40	30
<i>Jack</i>	40	1600, 1600	2000, 1500
	30	1500, 2000	1800, 1800

The payoff matrix (Jack, Jill).

What will Jack do? What will Jill do?

Dominant Strategies

The chosen actions are $\{40,40\}$, because each of Jack and Jill will choose to produce 40 litres, not 30.

Choosing 40 over 30 is a *dominant strategy* for each player, since whatever the other seller does each is better off by choosing 40 over 30 litres.

But this is frustrating: if they could collude or cooperate, they'd make \$1800 each, instead of \$1600. What is best collectively is not attainable individually. This is an example of the *Prisoner's Dilemma*.

Nash Equilibrium

Would Jack produce still more? Say 50 litres/week? If $Q = 40 + 50 = 90$ litres, then $P = \$30$, and Jack's profit would be $50 \times \$30 = \$1500 < \$1600$, so Jack has no incentive to produce more than 40 litres/week. Indeed, if both produce at 50 litres, each makes only \$1000.

$y^{Jack} = y^{Jill} = 40$ litres is a **Nash Equilibrium**: a situation in which each actor chooses her best strategy, given that the others have chosen their best strategies.

Named after John Nash, the Nobel laureate mathematician played by Russell Crowe in *A Beautiful Mind*.

http://images.countingdown.com/images/theater2/309230/media/309230_qt_h.mov

Payoff Matrix 2

		<i>Jill</i>	
		50	40
<i>Jack</i>	50	1000, 1000	1500, 1200
	40	1200, 1500	1600, 1600

The **Nash Equilibrium** at quantities $\{40,40\}$ (and $P = \$40/\text{litre}$) is shown by the **arrows**: any cell with no arrows leaving and only arrows into it is a Nash Equilibrium,

There may be one, several, or no Nash Equilibria.

This is not a Prisoner's Dilemma. Why? Because what is best individually is also best if they acted together.

Comparisons

So the duopolists produce at a rate (80 litres/week) less than competitive (120) but greater than monopolistic (60),

at a price (\$40/litre) greater than competitive (\$0), but lower than monopolistic (\$60).

Their total profits (\$3200/week) are less than monopolistic (\$3600), but greater than competitive (\$0).

A **Cournot duopoly** because the firms set the quantity, and the market (demand) determines the price;

in a **Bertrand duopoly** the firms set the price and the market determines the quantity.

3. The Prisoner's Dilemma

		<i>Kelly</i>	
		Spill	Mum
<i>Ned</i>	Spill	8, 8	0, 20
	Mum	20, 0	1, 1

Years of prison (Ned, Kelly).

The choices: Spill the beans to the cops, or keep Mum.

Nash Equilibrium = {Spill, Spill}, despite the longer sentences.

See also the *Tragedy of the Commons* in the Marks on-line reading.

<http://www.agsm.edu.au/~bobm/papers/ccp.pdf>

The Advertising P.D.

		<i>B & H</i>	
		Don't Advertise	Advertise
<i>Philip Morris</i>	Don't Advertise	\$4bn, \$4bn	\$2bn, \$5bn
	Advertise	\$5bn, \$2bn	\$3bn, \$3bn

Profits (Philip Morris, Benson & Hedges).

N.E. at {Advertise, Advertise}, despite the lower profits.

When tobacco advertising was banned on TV, tobacco firms' profits rose.

***n*-Person Prisoner's Dilemmas**

Examples?

- **the tragedy of the commons**
- **the common-pool oil-drilling problem**
- **cooperative pricing v. price wars**
- **tax compliance**
- **individual negotiation**
- **coal exports**
- **market development**
- **common property issues**
- **others?**

But People Do Cooperate

Why? The game is usually not played once, but many times.

If they only play once, then Jack and Jill, the Cournot duopolists, have no incentive not to cheat on their quotas of 30 litres.

But if each knows that they will interact every week, and that a single defection (to 40 litres) would result in an eternity of 40 litres (forever forgoing the extra \$200/week profit), this threat might support cooperation (30 litres/week).

In a *repeated PD*, so long as the discount rate is not too high, repetition will support cooperation.

4. Chicken! and Other Games

The notorious game of **Chicken!**, as played by young men in fast cars.

Here “**Bomber**” and “**Alien**” are matched.

		<i>Bomber</i>	
		<i>Veer</i>	<i>Straight</i>
<i>Alien</i>	<i>Veer</i>	Blah, Blah	Chicken!, Winner
	<i>Straight</i>	Winner, Chicken!	Death? Death?

Diagram illustrating the game of Chicken! with payoffs and strategic arrows:

- Alien Veer, Bomber Veer:** Blah, Blah
- Alien Veer, Bomber Straight:** Chicken!, Winner
- Alien Straight, Bomber Veer:** Winner, Chicken!
- Alien Straight, Bomber Straight:** Death? Death?

Red arrows indicate best responses for each player:

- From (Veer, Veer) to (Veer, Straight) for Bomber.
- From (Veer, Straight) to (Straight, Straight) for Bomber.
- From (Straight, Veer) to (Veer, Veer) for Alien.
- From (Straight, Straight) to (Veer, Straight) for Alien.

Green circles highlight the outcomes (Chicken!, Winner) and (Winner, Chicken!), which are Nash Equilibria.

No dominant strategies: what's best for one depends on the other's action.

N.E. where? Regrets?

The Macroeconomic Game: One Player Has a Dominant Strategy

		<i>RBA</i>	
		Low	High
<i>Gov't</i>	Balanced	3, 4	1, 3
	Deficit	4, 1	2, 2

Red arrows indicate dominant strategies: Gov't chooses Deficit (4 > 3 and 1 > 2), RBA chooses Low (4 > 2 and 3 > 1). The outcome (2, 2) is circled in green.

Players:

Gov't: fiscal policy (taxes, govt. expenditure)

RBA: monetary policy (interest rates)

Actions:

Gov't: either balanced budget or deficit

RBA: high or low interest rates

Preferences? (4 = best, 1 = worst):

Ex: The Macroeconomics Game

The RBA's best strategy depends on the Gov't's strategy. Dislikes inflation, High rates.

The Gov't prefers spending (and a budget deficit).

The RBA realises that {Deficit} is a dominant strategy for Gov't.

∴ RBA should choose {High}.

∴ Payoffs of (2,2), although {Balanced, Low} → (3,4) is jointly better.

Many countries have a loose fiscal policy and a tight monetary policy at {Deficit, High interest rates}.

5. Sequential Games

What if one player moves first?

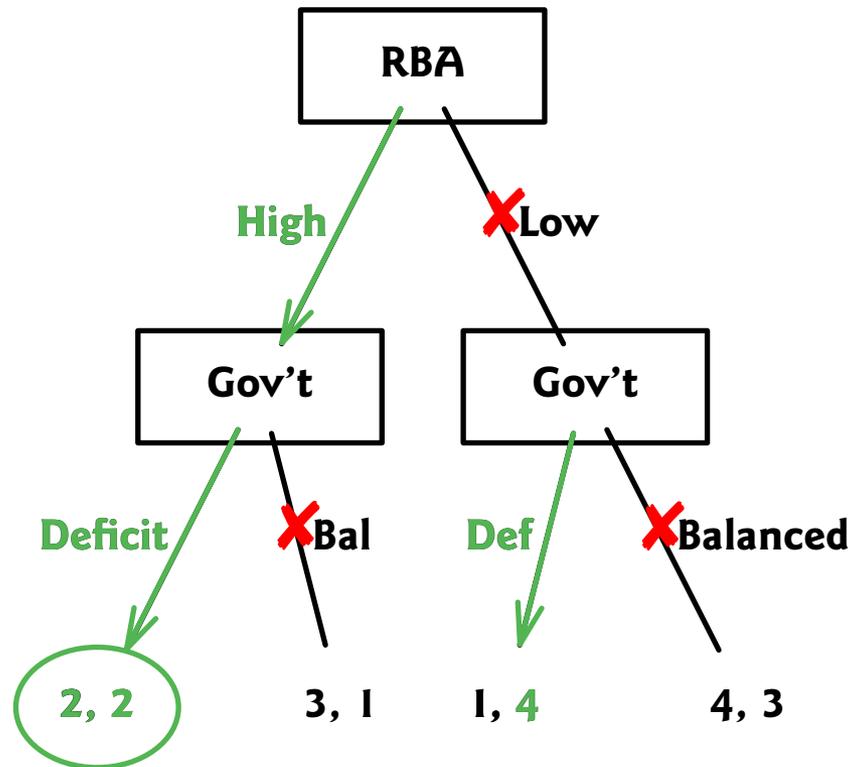
Use a *game tree*, in which the players, their actions, what they know (their information), and the timing of their actions are explicit.

Raises the possibility of First-Mover Advantage, or Second-Mover Advantage, and Threats and Promises, and Credibility, and Incomplete Information, and Screening and Signalling.

See *Strategic Game Theory for Managers* in Term 3.

What If The RBA Moves First in the Macro Game?

The game tree (4 = best, 1 = worst), (1st, 2nd mover):



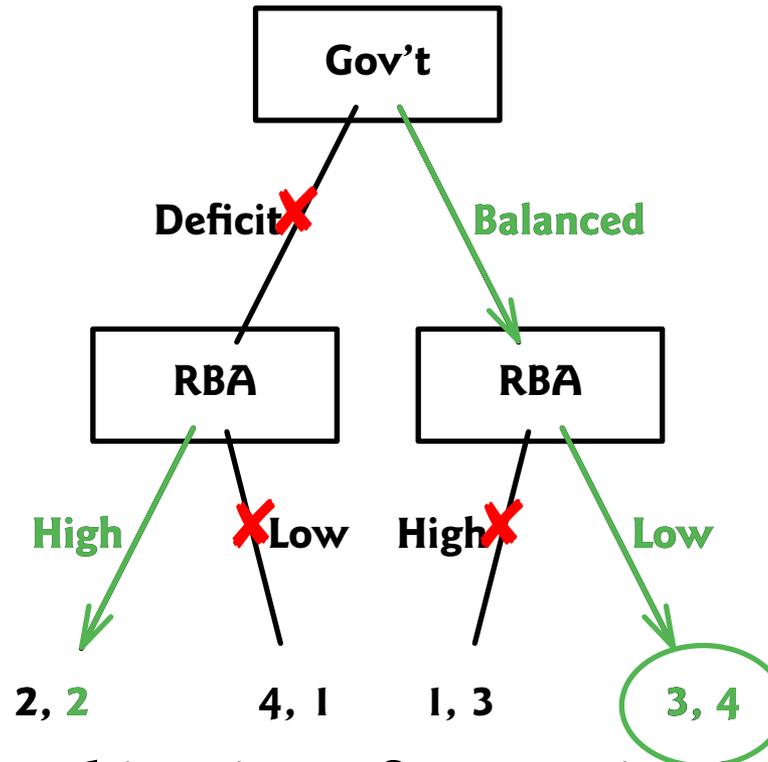
The RBA knows that the Gov't will go into {Deficit}, come what may, and so chooses {High} interest rates, yielding the RBA 2 instead of 1. As in the simultaneous game.

Pruning, or Rollback

- 1. From the bottom (final payoffs), go up the tree to the first parent decision nodes.**
- 2. Identify the best decision for the deciding player at each node.**
- 3. “Prune” all branches from the decision node in 2. Put payoffs at new end = best decision’s payoffs**
- 4. Do higher decision nodes remain?
If “no”, then finish.**
- 5. If “yes”, then go to step 1.**
- 6. For each player, the collection of best decisions at each decision node of that player → best strategies of that player.**

But if the Gov't moves first:

The game tree is:



The chosen combination of strategies is {Balanced, Low}: this is the **Rollback Equilibrium (R.E.)**, and, surprisingly, yields a better outcome for *both* players than does {Deficit, High}.

Boeing v. Airbus

Airbus and Boeing will develop a new commercial jet aircraft.

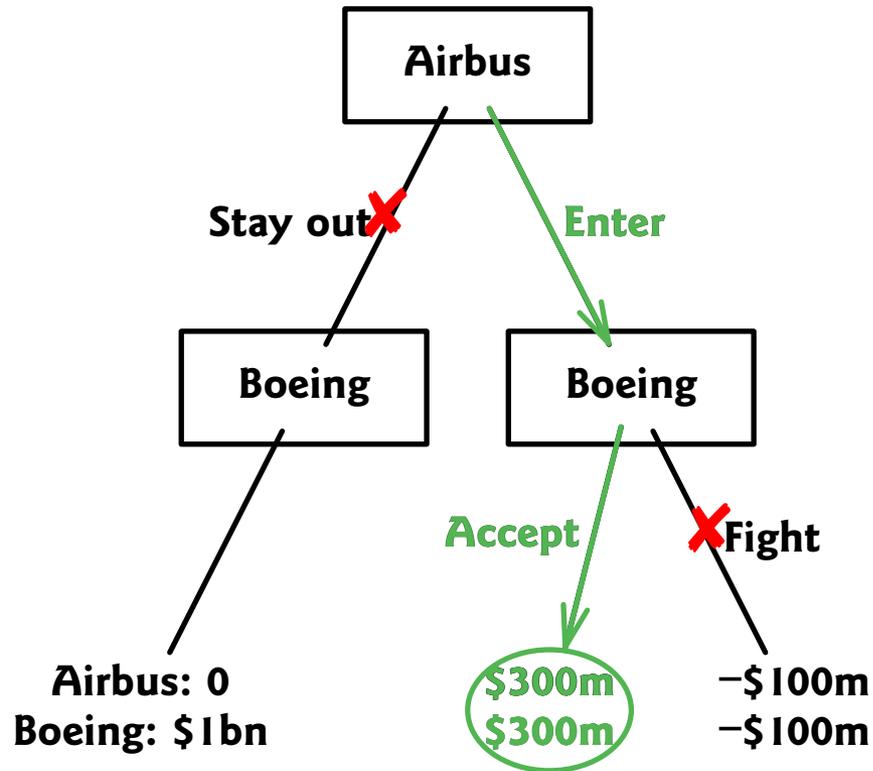
Boeing is ahead in development, and Airbus is considering whether to enter the market.

If Airbus stays out, then it earns zero profit, while Boeing enjoys a monopoly and earns a profit of \$1 billion.

If Airbus enters, then Boeing has to decide whether to accommodate Airbus peacefully, or to wage a price war.

**With peace, each firm will make a profit of \$300 m.
With a price war, each will lose \$100 m.**

A Game Tree



How should Boeing respond?

Questions

- 1. Draw the tree for this game. Use *rollback* (or backwards induction) to find the equilibrium.**
- 2. Why is Boeing unlikely to be happy about the equilibrium? What would it have preferred? Could it have made a credible threat to get Airbus to behave as it wanted?**
- 3. What if Boeing had moved first? Would there still have been a credibility problem with Price War? Explain.**

Summary

- 1. Oligopoly is a market structure between Perfect Competition and Monopoly, in which firms behave strategically.**
- 2. In a Cournot duopoly the two sellers of a homogeneous product choose quantities, and the market demand determines the price.**
- 3. Cooperation would lead to higher profits, but the logic of the once-off game is to cheat on agreed quotas → lower profits.**
- 4. Use Payoff Matrices for a simultaneous-move game and Game Trees for a sequential-move game.**

- 5. Use arrows in the Payoff Matrix to determine whether and where the Nash Equilibrium (in which each player does the best for herself, given that the other players are doing the best for themselves) is.**
- 6. A dominant strategy is an action that is best for you, no matter what the other player does.**
- 7. The Prisoner's Dilemma occurs when individual choices lead to a lower payoff than cooperative actions would.**
- 8. But repetition can overcome the once-off logic and result in cooperation.**

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- 9. Not all interactions have a single N.E. — some have none, some have several.**
 - 10. Can have 3×3 or larger payoff matrices.**
 - 11. Some market behaviours are illegal.**
 - 12. Rollback: look forward and reason back — to find the equilibrium of the sequential game.**

Appendix: Cartel v. Oligopoly

1. The *cartel* chooses $Q = y_1 + y_2$ to maximise its profit $\pi = \pi(y_1, y_2)$.

When production shares are equal ($y_1 = y_2$), then calculus ($\frac{\partial \pi}{\partial Q} = 0$) reveals that in this case with $P = 120 - Q$ and zero costs, then $y_1^* = y_2^* = 30$.

2. Each *oligopolist* chooses its output y_1 (or y_2) to maximise its profit $\pi_1 = \pi_1(y_1, y_2)$, but it has no control over the other firm's output y_2 .

Since the problem is symmetrical, assume $y_1 = y_2$, and calculus ($\frac{\partial \pi_1}{\partial y_1} = 0$) reveals that $y_1^* = y_2^* = 40$.