Basic Concepts In Project Appraisal

[C&B Ch. 2, 3; DoF Ch. 4; FP Ch. 3, 4, 5]

1.

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[C&B Ch. 2, 3; DoF Ch. 4; FP Ch. 3, 4, 5]

1. Which Investment Criterion?
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   Annual User Charge / Value On Completion / Annual Value / Annuities
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1. **Which Investment Criterion?**
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11. Capital Rationing

1. Which Investment Criterion?

[L 2.3]

Net Present Value:

$$NPV = \sum_{t=0}^{T} \frac{b(t) - c(t)}{(1 + r)^t} - K,$$

where NPV = net present value from project

$b(t)$ = benefits ($) received from project in year t

$c(t)$ = costs ($) of project in year t

$\dfrac{1}{1 + r(t)}$ = discount factor at interest rate r p.a.

T = lifetime of project

K = initial (capital) outlay at $t = 0$

Questions:

(These issues will take several lectures.)

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[C&B pp.41–53; DoF Ch. 4, App. III; FP Ch. 5]

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[C&B pp.41–53; DoF Ch. 4, App. III; FP Ch. 5]

3. Net Present Value (NPV).

3.1

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[C&B pp.41–53; DoF Ch. 4, App. III; FP Ch. 5]

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    - **3.1 Annual User Charge.**
    - **3.2**

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   - 3.2 **Value on Completion.**
   - 3.3

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Opportunity Cost again ...

The basis for decisons must be *opportunity cost*, or the value of options forgone.

Neither IRR nor B/C can be adequately used to choose between two mutually exclusive projects.

In general, we want to compare two (or more) projects and choose one (mutually exclusive).

Consider two projects, A and B.

Each costs $100 in year 0. Project A returns nothing in year 1, and $121 in final year 2. Project B returns $115 in final year 1, and nothing thereafter.

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- **At a discount rate of 5.2% pa the two projects are equally attractive (and have a positive NPV).**
- **At a discount rate of 10% pa Project A has an NPV of zero: its IRR is 10% pa. Why?**

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- **At a discount rate of 15% pa Project B has an NPV of zero: its IRR is 15% pa. Why?**

Consider two projects, A and B.

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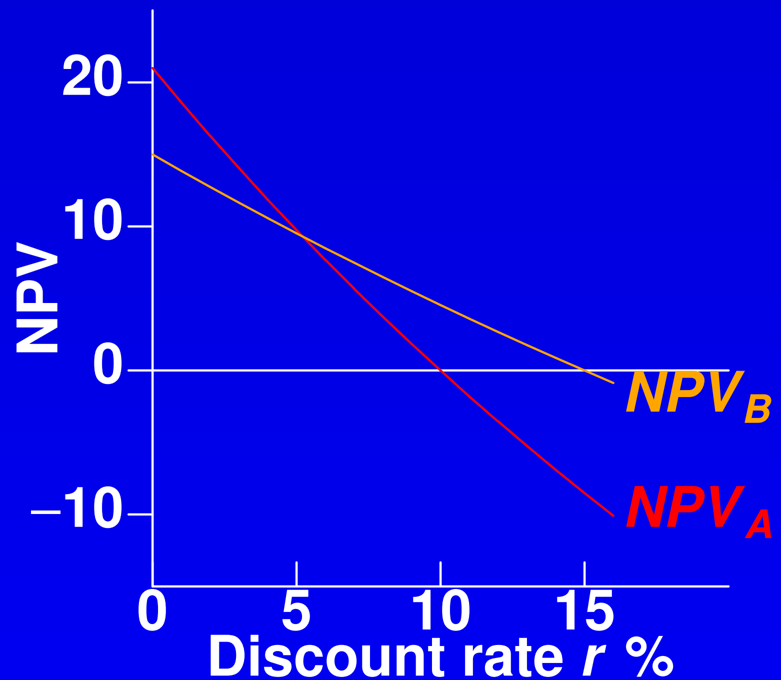
- **At a discount rate of 5.2% pa the two projects are equally attractive (and have a positive NPV).**
- **At a discount rate of 10% pa Project A has an NPV of zero: its IRR is 10% pa. Why? At a discount rate of 10% pa Project B has a positive NPV.**
- **At a discount rate of 15% pa Project B has an NPV of zero: its IRR is 15% pa. Why? At a discount rate of 15% pa Project A has a negative NPV.**

So, choose Project A if the market rate is less than 5.2%, or Project B otherwise, if the criterion is maximizing the NPV. Choose Project B if the criterion is maximizing IRR.



Find r_1 where two projects have equal NPV by solving for r_1:

$NPV_A(r_1) = NPV_B(r_1)$: $\rightarrow r_1 = 5.2\%$

$$\text{where } NPV_B = -100 + \frac{115}{1+r} \text{ and } NPV_A = -100 + \frac{121}{(1+r)^2}$$

3. Net Present Value

[C&B pp.41–43; DoF Ch. 4; L. 2.3; FP Ch. 5.1]

Calculate NPV (or NPB) of each project using r_m (the *appropriate* market rate or rates—they may vary through time—of return) (Using the formula on Lecture 3-2, above.)

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if NPV

> 0 then the project is OK

= 0 indifferent

< 0 then the project is not OK, because the return ("the appropriate market rate") is higher than the return from this project. The opportunity value is negative.

Many projects?

If there are many projects, mutually exclusive, and there is no budget constraint,

then rank by *positive* NPV > 0
and go with the largest NPV,
since this project maximises the size of the return.

Yes, if only 1 chosen.

No, if can choose several.

Three types of decision:

1.

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1. ***accept or reject:***

   accept if NPV > 0

   reject if NPV < 0

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   b.

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1. ***accept or reject:***

   accept if NPV > 0

   reject if NPV < 0

2. ***ranking* —**

   a. **If no capital budgeting, (or other rationing),**
   then accept all projects with NPV > 0

   b. **If there is *capital budgeting,* (See 11. below)**

   then rank: by B/C, not by NPV

< >

3.1 Annual User Charge (AUC)

***Concepts*:**

Opportunity Cost: **The opportunity cost of a project is what is forgone by undertaking the project — i.e. the value of resources in next-best use.**

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***Concepts*:**

Opportunity Cost: **The opportunity cost of a project is what is forgone by undertaking the project — i.e. the value of resources in next-best use.**

***Depreciation* (economic):** **The change (fall) in market value of an asset.**

Implicit rental cost: **The opportunity cost of holding (owning) an asset. (e.g. a machine)**

= the implicit rental cost

= the sum of:

the interest forgone on outlay +
depreciation +
any operating costs.

(Don't use straight-line depreciation: use annuity.)

Example of Annual User Charge (AUC):

Purchase of vehicle.

Bought for $2
Sold at $1 one year later.
i = 10% p.a.; costs $0.30 to run for one year

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Marginal cost:
How much more does it cost to produce an extra unit of output?

Average cost:
What does it cost per unit of output?

3.2 Value On Completion

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- **followed by net operating revenues x_t over the operating life of the project represented by L (t from $T+1$ to $T+L$)**

The NPV of the project can be assessed as:

$$NPV_0 = -x_0 - \frac{x_1}{1+r} - \frac{x_2}{(1+r)^2} - \cdots - \frac{x_T}{(1+r)^T} + \frac{x_{T+1}}{(1+r)^{T+1}} + \cdots + \frac{x_{T+L}}{(1+r)^{T+L}}$$

< >

VOC Criterion.

An equivalent but simpler method is to compute the *Value On Completion* (VOC):

$$VOC_T = x_0(1+r)^T + x_1(1+r)^{T-1} + \cdots + x_T$$

That is: accumulate forward your investment outlays at the cost of capital, to the last date (T) at which the completed project costs.

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That is: accumulate forward your investment outlays at the cost of capital, to the last date (T) at which the completed project costs.

Then: Compare VOC_T with NPV_T, where both evaluations refer to the same date.

So we compute:

$$NPV_T = \frac{x_{T+1}}{1+r} + \frac{x_{T+2}}{(1+r)^2} + \cdots + \frac{x_{T+L}}{(1+r)^L}$$

VOC Criterion:

Note:

$$NPV_0 = \frac{NPV_T - VOC_T}{(1+r)^T} > 0 \quad \text{if } NPV_T > VOC_T$$

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Accept the project if the VOC is less than or equal to the NPV of cash flows over the operating life of the project.

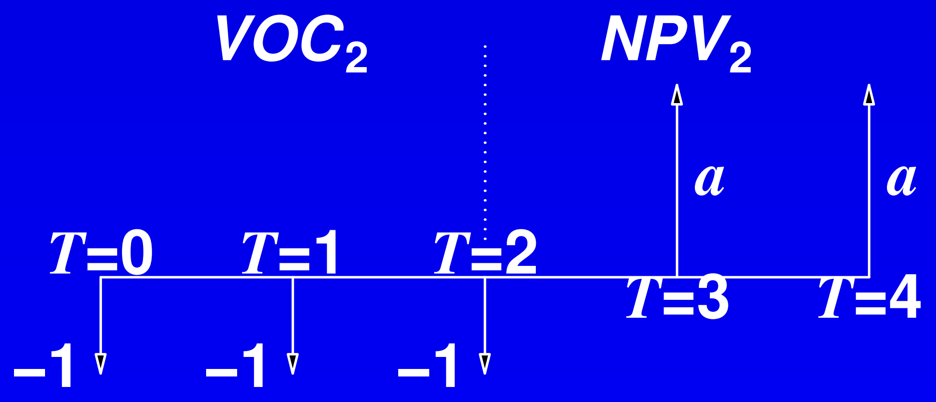
Moreover,
VOC = Direct Capital Outlays + Interest During Construction

Example 1 of VOC:

\$1 is outlaid at the beginning of each of 3 periods (T = 2). The asset operates for two years, yielding a net revenue stream of a (L = 2).

The discount rate r = 10% p.a.

Diagrammatically:



Calculating various values, forwards and back ...

$$NPV_0 = -1 - \frac{1}{(1.1)} - \frac{1}{(1.1)^2} + \frac{a}{(1.1)^3} + \frac{a}{(1.1)^4}$$

$$VOC_{T=2} = (1.1)^2 + 1.1 + 1 = 1.21 + 1.1 + 1 = 3.310$$

$$NPV_{T=2} = \frac{a}{1.1} + \frac{a}{(1.1)^2} = \frac{2.1a}{(1.1)^2} = 1.736a$$

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Is $VOC_{T=2} < NPV_{T=2}$?

Accept if 3.310 < 1.736a, or if a > 1.9072.

The annuity equivalent of $VOC_{T=2}$ = 3.310 is A = 1.9072. Hence the net revenue must exceed A, i.e., $a > A$.

Example 2 of the "VOC" Approach

The (early '90s) Very Fast Train (VFT):

Investment Outlay: $900m p.a. for each of 5 years

Cost of capital (assume 9.06% p.a.) (from database of CRIF, AGSM's Centre for Research in Finance)

Direct capital cost	**= $4.5 billion**
Value On Completion	**= $5.393 billion (includes return on capital)**
∴ Annual User Charge	**= $591 m p.a. (20-yr life)**

($5.393 bn is the present value of an annuity of $591 m over 20 years.)

The VFT continued ...

Operating and maintenance costs = $218m p.a.

∴ Total annual costs = $591 + $218 m = $809 m

Equivalent to 6¼ million passengers each paying $129 per trip.

NPV when first dollar is outlaid is zero.

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Equivalent to 6¼ million passengers each paying $129 per trip.

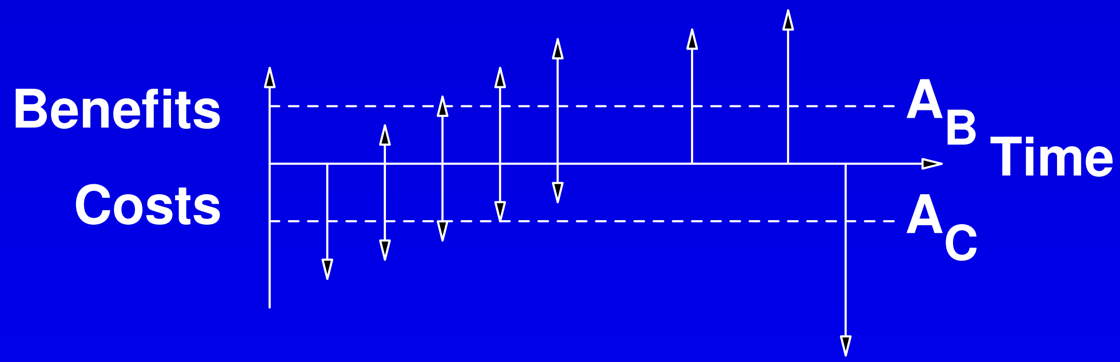
NPV when first dollar is outlaid is zero.

(So VOC equivalent to NPV (when costs & benefits are discounted to $T = 0$). Instead, the VOC takes costs & benefits to a date after investment is begun.)

3.3 Annual Value (Equivalent Annuities)
[C&B pp.30–31; DoF pp.46]

GPB $\rightarrow$ equivalent annuity A_B : PV $(A_B) =$ GPB

GPC $\rightarrow$ A_C : PV $(A_C) =$ GPC



accept/reject: $A_B - A_C \begin{cases} > 0 & \text{accept} \\ < 0 & \text{reject} \end{cases}$

rank by $(A_B - A_C)$

But AV: NPV

GPC $\rightarrow$ A_C

GPB $\rightarrow$ A_B

rank by $(A_B - A_C)$ $\sim$ NPV

Annuities and All That [C&B pp. 31–31]

$$FV = F_n = F_0(1+r)^n = F\,\frac{(1+r)^t - 1}{r}$$

where *FV* is the *future value* of an amount F_0 and *r* is the discount rate over *n* periods; where *F* is an *annuity* of over *t* periods.

When *n* is infinite, we have a *perpetuity*. In present value terms:

$$PV = \frac{F_n}{(1+r)^n}$$

$$PV = F\,\frac{1-(1+r)^{-t}}{r} \quad \text{annuity, and } PV = \frac{F}{r} \quad \text{perpetuity}$$

$$\therefore F = \frac{PV}{\dfrac{1-(1+r)^{-t}}{r}}$$

4. Internal Rate of Return

[C&B pp.45–49; DoF pp.114; L. 2.4; FP,Ch.5.2]

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Example: a cost of $1 now, a return of $1.10 in one period.

$$NPV = -1 + \frac{1.1}{1+i} = 0 \text{ at some } i\text{, the IRR.}$$

$\therefore$ **Internal rate of return = 10% =** i

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In general, $NPV = \sum_t \frac{x_t}{(1+i^*)^t} = 0 \rightarrow i^* = \text{IRR}.$

Rule:

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In general, $NPV = \sum_t \frac{x_t}{(1+i^*)^t} = 0 \rightarrow i^* = \text{IRR}.$

Rule: **undertake the project if its internal rate IRR exceeds the external yield (the market interest rate)**

But not if several and mutually exclusive ...

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Note: if projects are *mutually exclusive*, we cannot rank them by their internal rates of return.

Why?

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Why? Because the IRR is independent of the size or scale of the project: a minute project could have a much larger IRR than a project ten times bigger: *scale-independence*.

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Why? Because the IRR is independent of the size or scale of the project: a minute project could have a much larger IRR than a project ten times bigger: *scale-independence*.

IRR solves for the rate *r* which makes the present value of net benefits equal zero, or *GPB*(r^*) = *GPC*(r^*).

$$\text{IRR} = r^* : \sum_{t=0}^{T} \frac{b_t}{(1+r^*)^t} = \sum_{t=0}^{T} \frac{c_t}{(1+r^*)^t},$$

(where c_0 includes K_0)

IRR is compared to the Market Rate

We compare the IRR with r_m, the appropriate market rate:

if IRR > r_m then OK on opportunity cost grounds
if IRR < r_m then not OK
if IRR = r_m indifferent

If there exist many mutually exclusive projects, then rank in terms of their IRRs and go with the highest?

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if IRR $<$ r_m then not OK
if IRR $=$ r_m indifferent

If there exist many mutually exclusive projects, then rank in terms of their IRRs and go with the highest?

No.

But there are problems with IRR.
(See Luenberger in the Package.)

Criticisms of IRR:

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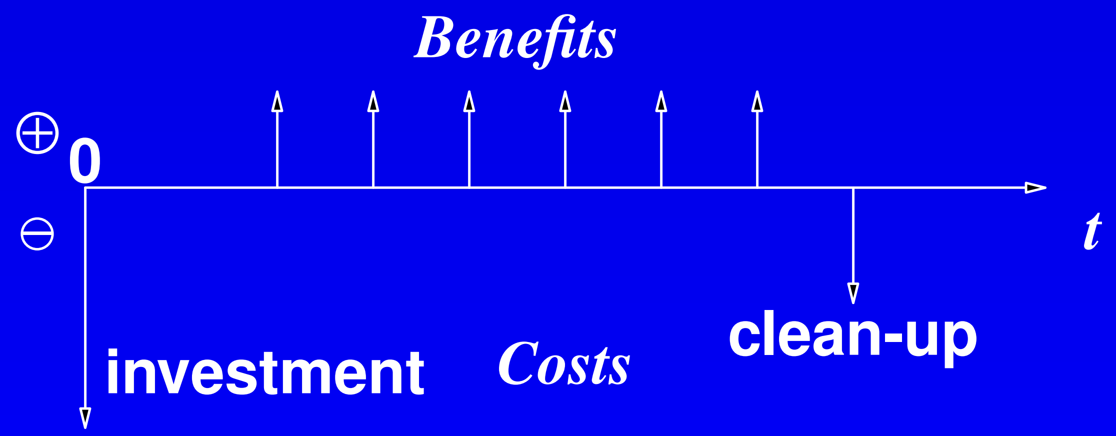
1. **Lack of uniqueness (may be several IRRs, *r* *).**
2. **Different time profiles of costs and benefits may result in ambiguous ranking.**

NB: Neither IRR nor B/C can be adequately used to choose between two mutually exclusive projects.



(a common profile) ⇒ IRR = 4% and 17%

5. Benefit/Cost Ratio

[C&B pp.43–44; DoF pp. 112; FP Ch. 5.5]

Calculate the ratio of $\dfrac{\text{p. v. of benefits}}{\text{p. v. of costs}} = \dfrac{B}{C}$ **or**

$$\frac{\displaystyle\sum_{t=0}^{T} \frac{b_t}{(1+r_m)^t}}{\displaystyle\sum_{t=0}^{T} \frac{c_t}{(1+r_m)^t} + K} = \frac{B}{C}$$

If $\frac{B}{C} > 1$**, then the project is OK, according to this criterion.** $\Leftrightarrow$ **NPV ≡ B – C > 0.**

Mutual Exclusivity [C&B pp.47–49]

If there are many mutually exclusive projects, then rank in terms of $\frac{B}{C}$ ratio and choose the project with the largest ratio?" No — it's scale independent.

This doesn't guarantee that NPV is maximised, and so the best project chosen.

There is, however, a rôle for B/C when there is *capital rationing*. (See 11. below.)

5.1 Net Benefit Investment Ratio, NBIR, or Profitability Ratio [FP pp.80–82; C&B p.50]

$$NBIR = \frac{\sum_{t=0}^{T} \frac{B_t - OC_t}{(1+i)^t}}{\sum_{t=0}^{T} \frac{IC_t}{(1+i)^t}}$$

where:

- OC_t are the project's operating costs in period t,
- IC_t are the project's investment costs in period t,
- B_t are the benefits in period t,
- i is the appropriate discount rate.

NBIR separates the project's operating costs and investment costs, to enable calculation of the net operating profit per present-value dollar invested.

6. Payback Period

[DoF pp. 115; FP Ch. 4.10.2]

$$\frac{K}{b_t}, \text{ implicitly } r = 0$$

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— not necessarily consistent with NPV

— bias towards projects with front-end returns (i.e., if recover costs in $t \leq \tau$ then OK)

7. Inflation

[C&B pp.64–66; DoF pp. 52; L. 2.6; FP Ch.4.9]

(Inflation: An increase in the general price level)

NPV is invariant to the inflation rate!

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Let R = nominal interest rate
i = real interest rate
g = rate of inflation of all prices

$$1 + R = (1 + i)(1 + g) = 1 + ig + i + g$$

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1. **In NPV analysis we can project price increases into the future and use nominal interest rate R (current dollars)**
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looking ahead: expected inflation rate?

1. **In NPV analysis we can project price increases into the future and use nominal interest rate R (current dollars)**

   or

2. **Can forget about future general price increases and use real interest rate i. (adjusted for inflation)**
3. **We get the same answer in both cases.**

Example with Inflation:

Cost of \$1 now, a real return of \$1.10 in one period;
$\therefore$ nominal return $\approx 1.10 \times (1+g)$.

$$NPV \approx -1 + \frac{1.1(1+g)}{1+R} \quad \text{(nominal)}$$

$$= -1 + \frac{1.1(1+g)}{(1+i)(1+g)}$$

$$= -1 + \frac{1.1}{1+i} \quad \text{(real)}$$

If real interest rate i = 10%p.a., then NPV = 0 regardless of inflation rate g.

$\therefore$ NPV is *not* a function of the inflation rate!

8. Income Tax

[FP Ch. 3.6]

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- **i is used to discount pre-tax cash flows**
- **$i(1-\tau_i)$ to discount after-tax cash flows**

Example with Taxes: Taxi Plate —

Net cash flow = \$10,000 p.a. pre-tax
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$$\therefore \text{NPV (post-tax)} = \frac{\$10,000\,(1-\tau_c)}{0.1\,(1-\tau_i)}$$

$$= \frac{\$10,000\,(1-0.5)}{0.1\,(1-0.5)}$$

= \$100,000 (perpetuity)

What if the two tax rates are not equal?

Classical tax system versus Imputation system

Suppose: Nominal interest rate *R*	= 15% p.a.
Expected inflation rate *g*	= 10% p.a.
∴ Real interest rate *i*	≈ 5% p.a.
Risk premium on equity	= 7% p.a.
Corporate tax rate (nominal effective)	= 39%
Debt:equity split	= 50:50

Classical (pre-1987) cost of capital:

Pre-tax (nominal) basis: ½ debt + ½ equity

$$15\%\frac{1}{2} + \frac{15\% + 7\%}{1 - 39\%}\frac{1}{2}$$

= 25.5% nominal

since the required 22% nominal return on equity is grossed up by the corporate tax factor (1 – 39%).

∴ Pre-tax (real) basis: 25.5% – 10% = 15.5% real

With the Australian tax imputation system for dividends (post-1987):

Company tax at the rate of 39% was effectively abolished for Australian taxpaying owners of Australian companies, so that the weighted average discount rate (nominal) is:

$$15\% \tfrac{1}{2} + (15\% + 7\%) \tfrac{1}{2} = 18.5\% \text{ nominal}$$

$\rightarrow$ **8.5% real (for such a personal investor.)**

9. Discount Rates for Private (and Public) Sector Projects:

Four concepts: [C&B pp.112–113, 221–229; DoF pp.57]

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   debt — borrowing
   equity — owning
   (See the example in 8 on Tax above.)

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5. ***Special Cases***

9.1 Social Rate of Time Preference (SRTP)

Society's preference for present consumption versus future consumption.

***or*:**

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Society's preference for present consumption versus future consumption.

***or*: the additional future consumption required to exactly compensate for postponement of a unit of present consumption now.**

SRTP $\neq$ individual's RTP necessarily

Estimating SRTP:

The exchange of government bonds → bond rate

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— long term (up to 50 years)

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At the margin, the return is equal to all (MRS), say, 10% p.a. nominal (or lower?)

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At the margin, the return is equal to all (MRS), say, 10% p.a. nominal (or lower?)

Adjust for inflation: 10 – 3 = 7% p.a. (real)
Adjust for taxation: $7(1 - \frac{1}{3}) = 4.7\%$ p.a.

But: some seek higher returns, while some invest nothing.

Net: is SRTP an upper bound?

9.2 Social Opportunity Cost of Capital (SOCC)

The return on the investment that is displaced by the marginal project.

With fully competitive markets: SOCC ~ SRTP
But with tax etc.: SOCC > SRTP

Net benefits are consumption, not investment.

Estimate:

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The return on the investment that is displaced by the marginal project.

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But with tax etc.: SOCC > SRTP

Net benefits are consumption, not investment.

Estimate: bond rate of 10% p.a. nominal, before tax
+ risk premium 2%

∴ real = 12% – expected inflation of 3%

∴ real = 9%

tax adjustment of $9(1 - \frac{1}{3}) = 6\%$ effective

9.3 Project-Specific Rates

Use CAPM to get market premium (2.1 – 7.9%) (Use AGSM Centre for Research In Finance (CRIF) database.)

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9.4 Cost of Funds

If the government is borrowing, then the long-term bond rate.

If private borrowing, see examples in 8 on Tax above.

A Bias Towards Government Projects?

If we use the lower SRTP, then

- **government**

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$\rightarrow$ **infrastructure bonds (lower discount rate) for private projects in order to lower the private cost, $\rightarrow$ PPPs, Public-Private Partnerships**

$\rightarrow$ **SOCC or project-specific rates for the government**

DoF: 8% p.a. as benchmark (real)
= 2% margin + 6% risk-free

(Probably too high for a risk-free rate now, 2006.)

SRTP versus SOCC

	Time preference SRTP	*Opportunity cost* SOCC
Goal	Achieve a preferred flow of net benefits over time.	Increase net income to society.
Time span	Long term—as long as individuals plan (say, one to two generations)	Short term—as long as the life of displaced private investment (say, up to 15 years)
Estimation can start from ...	... government bonds	... government bonds plus a risk premium
"Typical" real rates, after adjustment for taxation	4% to 7%	Above 7%

(from Sinden & Thampapillai, p.134)

10. Consistency of Horizon Choice

[C&B pp.51–53; L p. 25,26,29]

The plantation costs $1 to establish.
Two choices:

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A. cut it down after one year to receive $2, or

B. wait until the end of the second year and reap $3.

Compare the two projects using NPV and IRR:

Net Present Value @ 10% p.a.:

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- **B. wait until the end of the second year and reap $3.**

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Net Present Value @ 10% p.a.:

- **A. NPV of cutting sooner is $0.82 =** $-1 + \frac{2}{1.1}$
- **B. NPV of cutting later is $1.48 =** $-1 + \frac{3}{1.1^2}$

So (B) *later* looks more attractive using NPV.

Internal Rate of Return (irr):

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A. **Solve $-1 + 2c = 0$ where $c = \dfrac{1}{1 + irr}$, then *irr* = 1.0 or 100%.**

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A. Solve $-1 + 2c = 0$ where $c = \dfrac{1}{1 + irr}$, then $irr = 1.0$ or 100%.

B. Solve $-1 + 3c^2 = 0$, so $irr = \sqrt{3} - 1 = 0.73$ or 73%.

So (A) *earlier* looks more attractive using IRR.

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B. Solve $-1 + 3c^2 = 0$, so *irr* = $\sqrt{3} - 1 = 0.73$ or 73%.

So (A) *earlier* looks more attractive using IRR.

But the time horizon isn't OK:

A takes only 1 year, versus 2 years for B.

If we repeat (A) twice, its NPV is given by:

NPV(A twice) = $0.82\left(1 + \frac{1}{1.1}\right) = 1.5654 > 1.48$ (of B)

So choose A (= *earlier*) repeated.

Continues ...

The IRR of twice A is unchanged: 100% per year for 2 years.

Or: Solve $-1 + 2c + c(-1 + 2c) = 0$,
gives $c = -1$ or ½,
gives *irr* = 1.0 or 100%, (ignoring the negative root for *c*).

Another reason for choosing project A:

if the projects are repeated and the principal is reinvested, then (A) leads to doubling of principal every year, whereas (B) only grows at $\sqrt{3} = 1.73$ every year.

Note that the growth rate with reinvestment of principal = 1 + *irr* always.

11. Capital Rationing

[C&B pp.50–51; FP Ch. 5-App. 1; S&W, Ch. 6.2]

Aim of agency or firm: to maximise its financial surplus, subject to its capital budget.

e.g.

11. Capital Rationing

[C&B pp.50–51; FP Ch. 5-App. 1; S&W, Ch. 6.2]

Aim of agency or firm: to maximise its financial surplus, subject to its capital budget.

e.g. A public agency may spend up to $1.8m in year 0. Four independent, *divisible* (which means that fractional (≤ 1) projects are possible) projects are under consideration:

Project	*Cost in year 0 (to be paid from capital ration)* **$m**	*Net returns per year: year 1 onwards* **$m**	*Life of project (number of years in which net returns occur)*	*Gross Present Benefit*
A	**1.0**	**0.14**	**20**	**1.19**
B	**1.0**	**0.13**	**50**	**1.29**
C	**2.0**	**1.00**	**3**	**2.49**
D	**1.0**	**0.25**	**8**	**1.33**

Capital Rationing

Choose projects with the highest present value of capital used. At *r* = 10% pa, the NPVs are calculated:

Project	*NPV of project in year 0* **$m**	*Present value per unit of capital* **$**	*Internal rate of return* **(%)**
A	**0.19**	**0.19**	**13**
B	**0.29**	**0.29**	**13**
C	**0.49**	**0.24**	**23**
D	**0.33**	**0.33**	**19**

$$= NPV/K$$

$$= \frac{B - C - K}{K}$$

= a *B*/*C* ratio

Example of capital rationing:

The most efficient investment is $1.0m in Project D and the remaining $0.8m in Project B. This produces a total NPV of $0.56m (= $0.33 + 0.8 × $0.29).

If another $1 is available for investment, then we should invest further in Project B, increasing the NPV of the agency's financial surplus by $0.29.

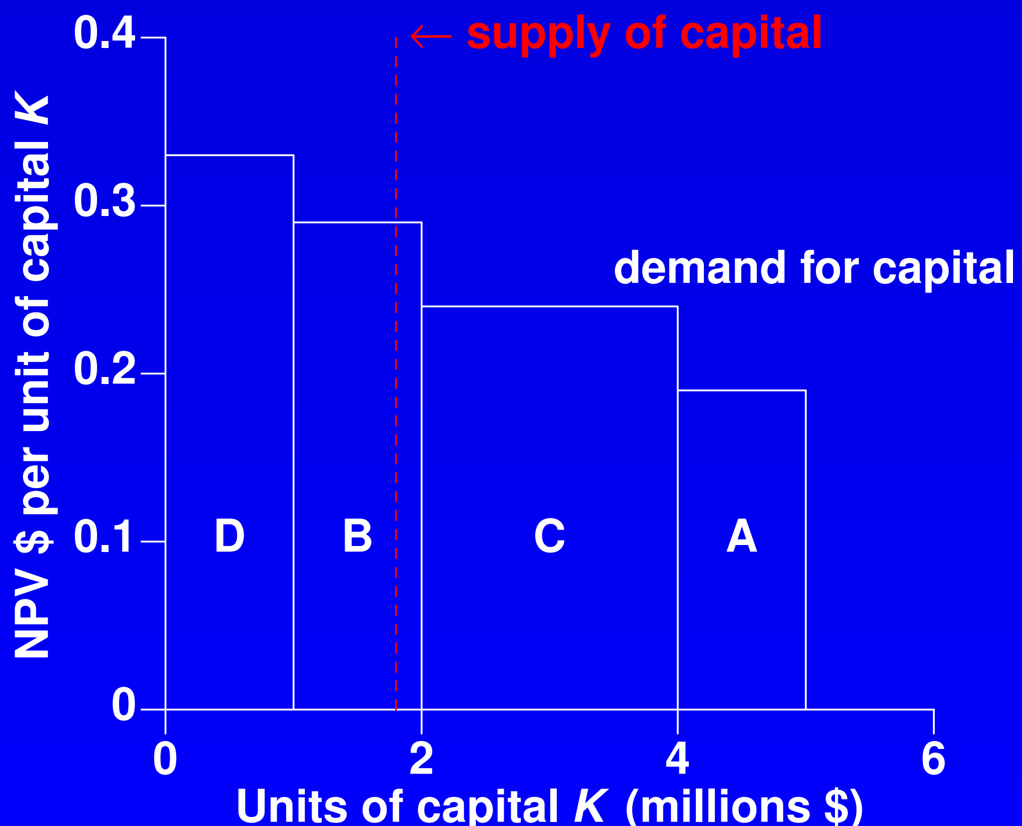
So one unit of capital, of a nominal value of $1, has at the margin a value of $1.29 with capital rationing.

The marginal opportunity cost of capital is $1.29.

We refer to this as a *shadow price* (greater than the nominal price of $1 because of the capital constraint).

Capital Rationing — Diagram

We can interpret this constraint with a simple supply–demand-for-capital figure: [S&W, Fig. 6.1]



Areas = NPV of projects.

Summary

[DoF p.54]

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- **Where *alternative projects* are under consideration, the project which maximises NPV should be selected.**

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- **Other decision rules, such as the *benefit/cost ratio* and the *internal rate of return* (IRR) may be included in the analysis alongside the NPV criterion, but, since these rules can be misleading except in restricted circumstances, they should not be used instead of the NPV criterion.**

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- **Do not confuse *real and nominal prices and discount rates* in the same analysis: where the analysis is in terms of nominal prices, the discount rate must be adjusted up to account for the expected inflation rate.**

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- Conclusion:

  The NPV rule should be the primary basis for decision-making, and should *always* be included.

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