Shadow Pricing, Direct Price Effects 1. Shadow Pricing

[C&B Ch. 5; DoF Ch. 3; FP Ch. 1.6, 6; S&W Ch. 8]

The NPV formula can be written as

$$NPV = \sum_{t} \frac{b(t) - c(t)}{(1+r)^t}$$
 where $b(t) = \sum_{i} b_i p_i = b_0 p_0 + b_1 p_1 + b_2 p_2 + \cdots$ and $c(t) = \sum_{j} c_j p_j = \cdots$

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Now, we assume here that we have the quantities b_i benefits and c_i costs.

What of the prices p_i and p_j ?

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→ shadow prices.

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- 5. of a tariff (a tax on imports).

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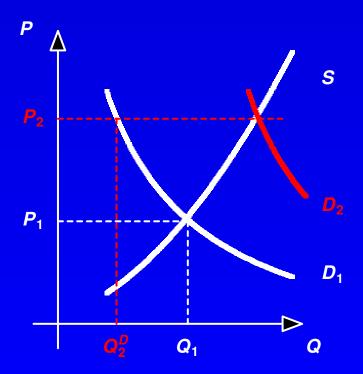
C&B's Pricing Rule. [C&B p.93]

Item to be valued	Valued at equilibrium point on a:	
	Demand Curve	Supply Curve
Output	Satisfies additional demand	Satisfies existing demand from alternative source
Input	Sourced from an alternative market source	Sourced from additional supply

(See also the Table on p.30 of the DoF *Handbook*.)

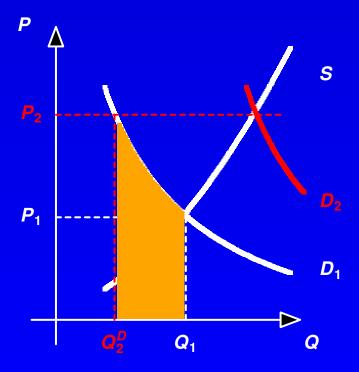
The Area under the Demand Curve is the Value of that Quantity

A demand curve shows the maximum the buyer is prepared to pay for each unit bought. As the price rises, buyers choose to buy less. They forgo the value to them of buying and using the forgone quantity: area = value forgone, as price $P_1 \rightarrow P_2$.



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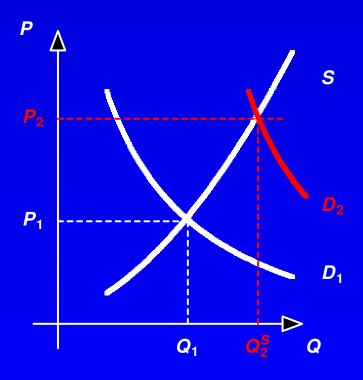
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Remember: the value of the last unit bought (the marginal value to the buyer) = the price of that unit.

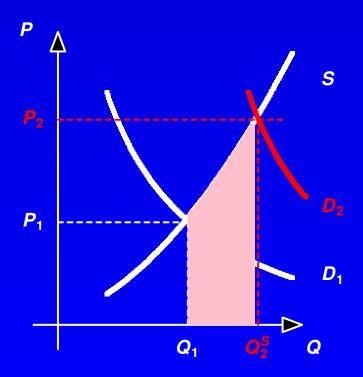
The Area under the Supply Curve is the Cost of that Quantity

To a firm in a competitive market, its supply curve shows the cost of supplying each unit. A higher price induces more units to be offered for sale: the area under the supply curve is the total cost of supplying those units, as price $P_1 \rightarrow P_2$.



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Remember: the marginal cost of supply (the cost of supplying the last unit sold) = the price of that unit.

Q:

Q: A remote electricity-generation project pays \$1/litre for its fuel oil, the costliest input to the project. The FA (financial appraisal) gives an NPV close to zero, but there is a tax on the fuel oil of 45¢/litre. What is the shadow price of fuel oil, to be used in the CBA NPV?

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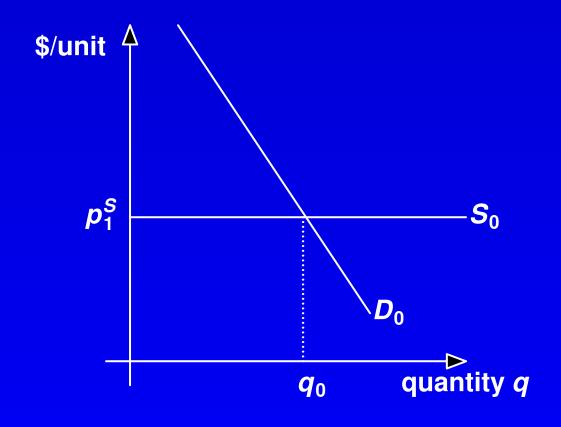
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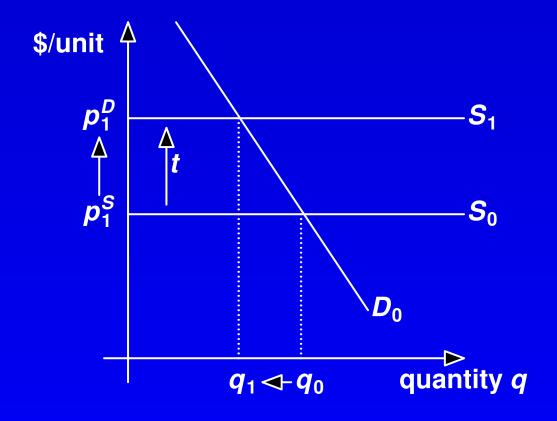
A (specific) tax on a good supplied in a competitive market:

- places a wedge between the marginal cost (supply) and price (demand)
- : the single (equilibrium) price p_0^S can no longer represent both valuation and cost
 - Suppose the good is an input into a project:

A Tax (cont.) — Infinitely elastic supply

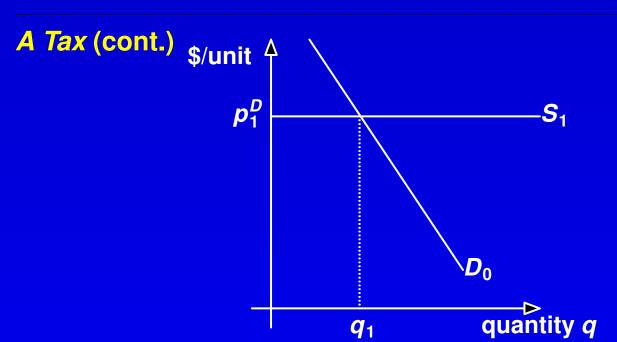


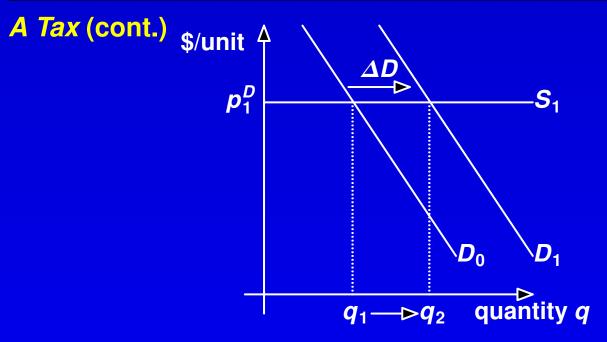
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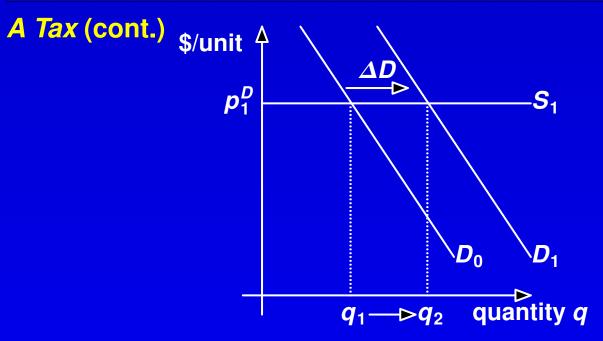


Because of the tax, the (tax-inclusive) demand price p_1^D is greater than the (tax-exclusive) supply price p_0^S .

The diagram shows the Tax Revenue and the Dead-Weight Loss as the tax is imposed, pushing up the effective supply, and reducing the quantity demanded, from q_0 to q_1 .

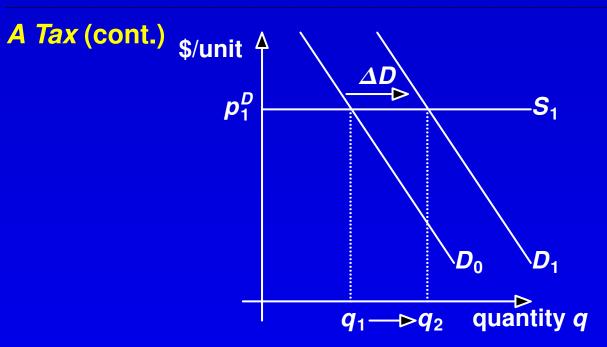




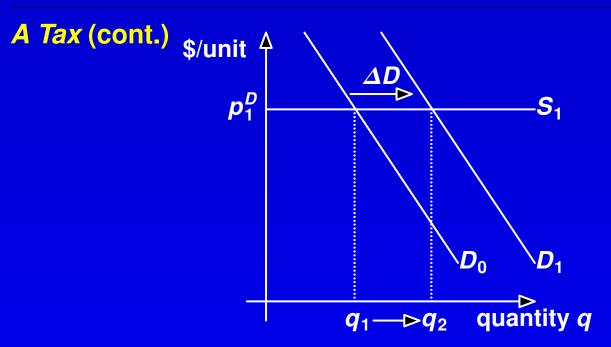


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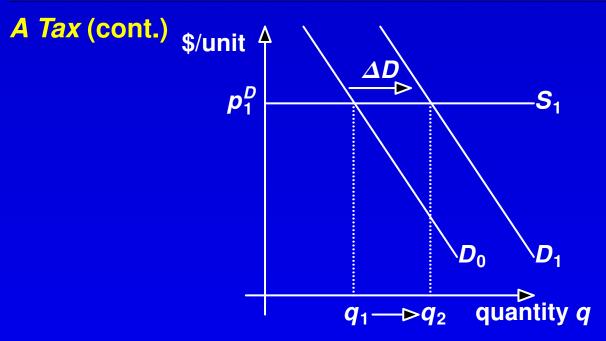
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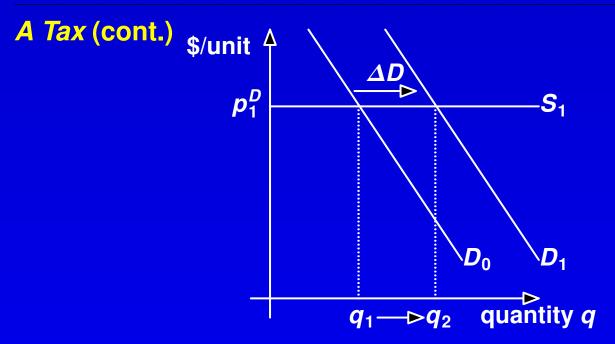
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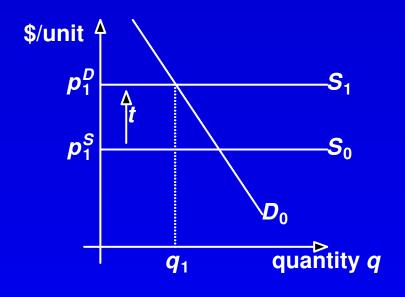
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- No, in general, but it depends on the purpose of the tax (i.e., revenue or "green" tax?).

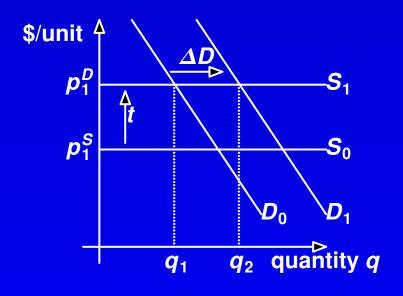
A Tax (cont.) —

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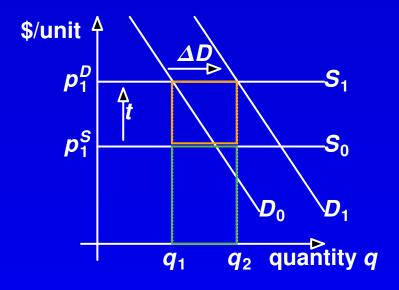
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Consumers value the increase in demand (ΔD) at the tax-inclusive price p_1^D .

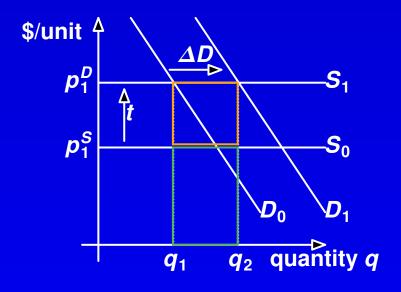
Suppliers' price is unchanged at p_1^S .

Shadow price = $p_1^D - t = p_1^S$ = unchanging tax-exclusive price, p_1^S .

The tax revenue (\square) is a transfer, and so changes in the tax revenue (changes in a transfer) are not changes in cost (\square).

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This is only the case if there is *no effect on existing purchasers of output*, since there is no increase in price with perfectly elastic supply.

 \therefore the ex-tax price p_1^S is the shadow price.

[C&B pp. 92–96; FP Ch. 2.2.2, 2.3.2; DoF 3.4]

Q:

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Q: There is a local market for irrigation water. The going price is \$50/megalitre. A new cotton farm is planned, but its size and thirst for water are such that the going price of water will rise to \$60/megalitre, given its demand of 10,000 megalitres/year. At the lower price the NPV of the project is positive, but at the higher price negative. At the higher price the incumbent users cut their consumption by 1000 megalitres/year.

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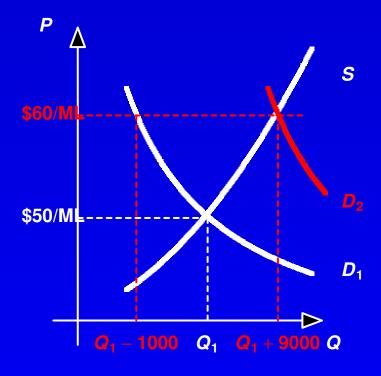
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- A: The shadow price is between \$50 and \$60/megalitre, say \$55 (assuming linear supply and demand curves). The existing users bear a cost of $$55 \times 1000 = $55,000/year$ for the water they can no longer afford (the *displaced* water).

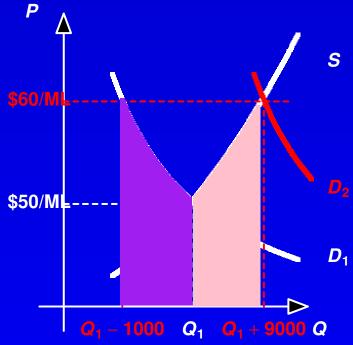
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(Not to scale.)

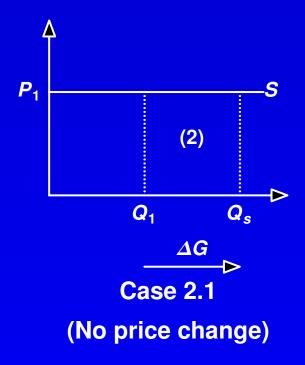


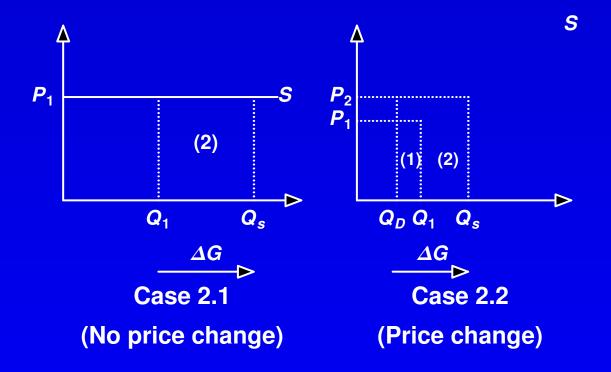
(Not to scale.)



value of displaced water = \$55,000 cost of supplying incremental water = \$495,000 \therefore total cost = \$550,000 $\rightarrow P_S$ = \$55

For a FA, cost = $$60 \times 10,000 = $600,000$ For a CBA, cost = $$55 \times 10,000 = $550,000$





Case 2.1: (No price change)
resource opportunity cost = total social costs for
increased factor supply (2) $= P_1 \cdot \Delta G$

Case 2.2: (Price change)

resource opportunity cost = total social costs $P_1 \cdot \Delta G \qquad \text{for increased factor supply (2)}$

 + value of reduced use of inputs in the rest of society as a response to higher prices, or opportunity costs. (1)

$$P_1 \cdot \Delta G < \text{Area } [(1) + (2)] < P_2 \cdot \Delta G$$

$$\therefore P_1 < P_s < P_2$$

 \rightarrow P_s is the "effective or shadow price": $P_s \cdot \Delta G$ is the resource cost = area (1) + area (2)

Note: be conservative

if NPV > 0 with $P_2 \sim \text{cost}$

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if NPV > 0 with $P_2 \sim \text{cost}$ then GO

if NPV < 0 with $P_1 \sim \text{cost}$ then STOP

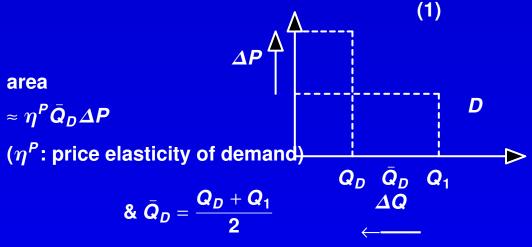
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$$\label{eq:problem} \begin{array}{l} \text{if NPV} > 0 \text{ with } P_2 \sim \text{cost} & \text{then GO} \\ \text{if NPV} < 0 \text{ with } P_1 \sim \text{cost} & \text{then STOP} \\ \\ NPV\left(P_2\right) < 0 & \\ NPV\left(P_1\right) > 0 & \text{then must find } P_s \\ NPV\left(P_s\right) ? & \end{array}$$

The point is so avoid the cost and effort of deriving a better estimate of the shadow price P_s if it won't make any difference to the decision.

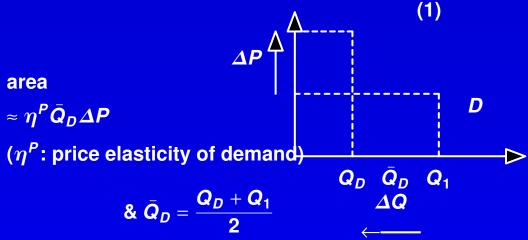
Some Equations and Harberger's Method

Area under Demand Curve

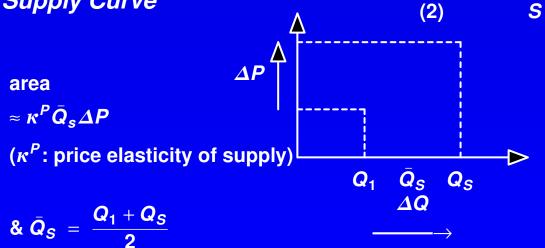


Some Equations and Harberger's Method

Area under Demand Curve



Area under Supply Curve



Case 2.2: Prices change. (NFX: Not For Exam)

From above, social cost (1) + (2) = $P_s \cdot \Delta G$

$$= \Delta P \left(\eta \bar{Q}_D + \kappa \bar{Q}_s \right)$$

$$\therefore P_{s} = \frac{\Delta P \left(\eta \bar{Q}_{D} + \kappa \bar{Q}_{S} \right)}{\Delta G} \text{ shadow price}$$

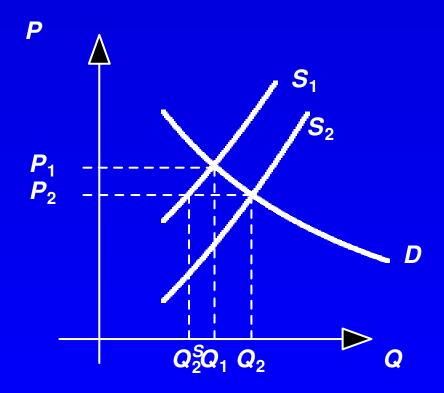
$$= \frac{\Delta P \left(\eta \frac{Q_{1} + Q_{D}}{2} + \kappa \frac{Q_{1} + Q_{S}}{2} \right)}{\Delta G}$$

$$= \frac{\Delta P \eta \left(Q_{1} + \frac{Q_{D}}{2} + \frac{Q_{S}}{2} \right)}{\Delta G} \text{ (if } \eta = \kappa \text{)}$$

— a means of obtaining the shadow price P_s from Q_1 , ΔP , η , κ , Q_D , Q_S , and ΔG .

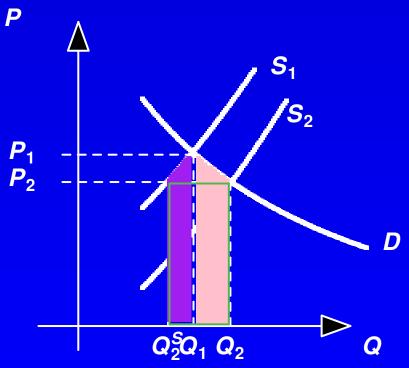
The Project's Output Drives Prices Down

The project produces an output which is sold: FA benefit = revenue = selling price × quantity. With no change in price, same with CBA. If the output price falls, some marginal producers cut back or cease: a benefit. FA revenue is too low.



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value of the displaced production value of the incremental production P_1 < shadow price < P_2

Q:

Q: In the cotton-farm example, assume that the prices of \$50 (before) and \$60/megalitre (after) include a tax of \$4/megalitre, to raise revenue. What now is the shadow cost of water to the new farm?

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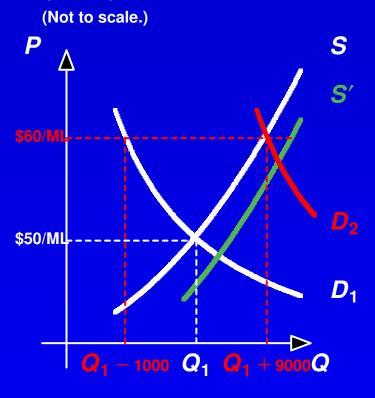
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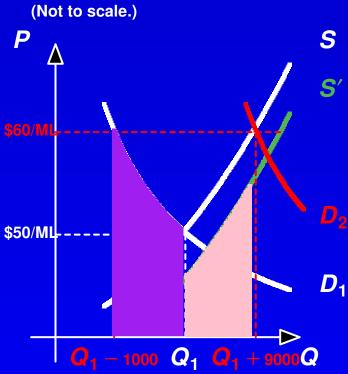
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The shadow cost of the *incremental* water is between \$46 and \$56/megalitre (the tax-exclusive prices), since that's what the suppliers of water receive to induce them to increase supply; say $$51 \times 9000 \text{ megalitres/year} = $459,000/year.$

- ∴ Total shadow cost = \$55,000 + \$459,000 = 514,000/year.
- ∴ Shadow price = \$51.40/megalitre

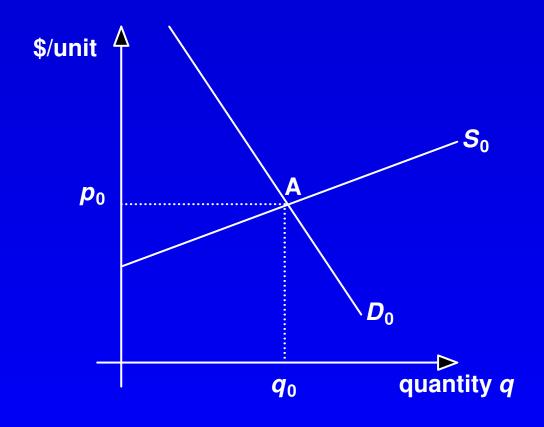




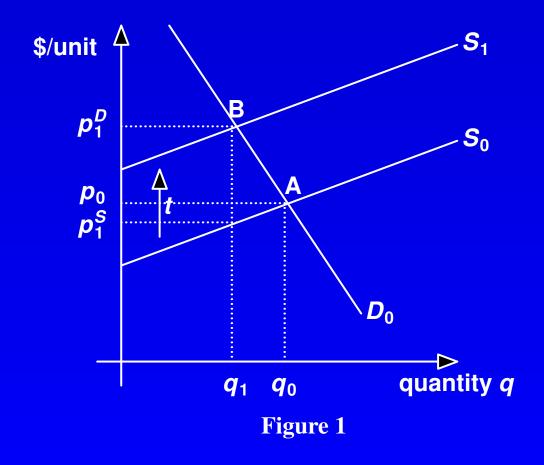
value of displaced water = \$55,000 (still) cost of supplying incremental water = \$459,000 \therefore total cost = \$514,000 \rightarrow P_S = \$51.40

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Elastic supply



Elastic supply



In Figure 1 above:

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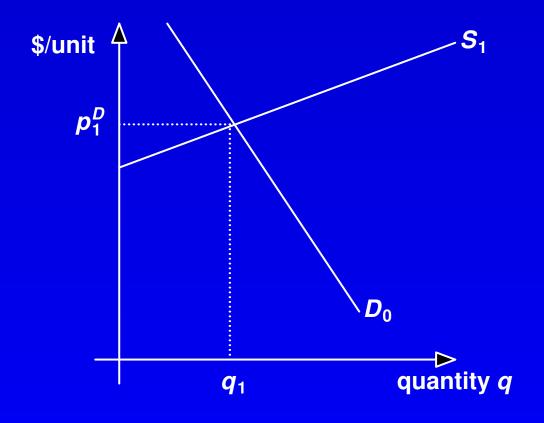
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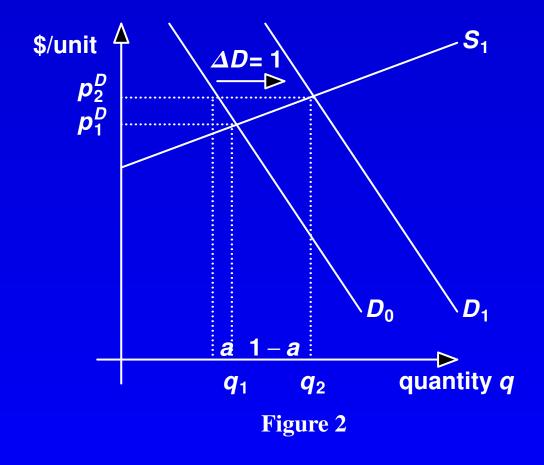
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- Existing consumers value the reduction in $m{a}$ at the tax-inclusive price $m{p}_2^D$ that they pay
- ... unit social cost = a(gross-of-tax price) + (1-a)(net-of-tax price)= $a p_2^D + (1-a)(p_2^D - t) = shadow price$

Hence shadow price = a weighted average of the tax-inclusive and tax-exclusive prices, p_2 and $p_2 - t$, respectively.

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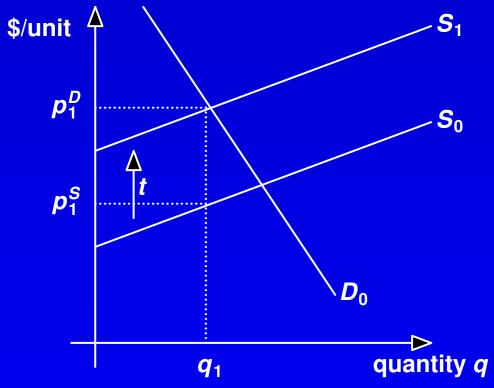
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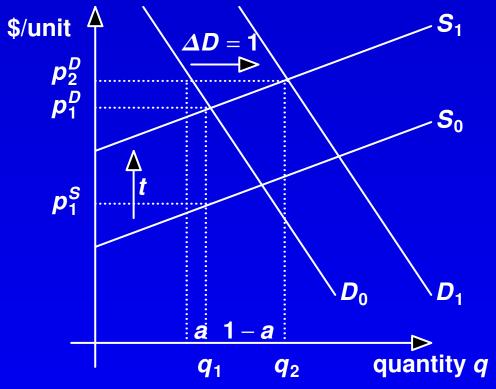
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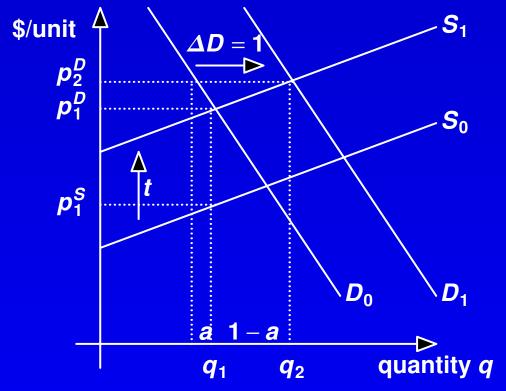
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- for an increased demand for the input
- also for an increase in supply of the good if the project results in more of the good

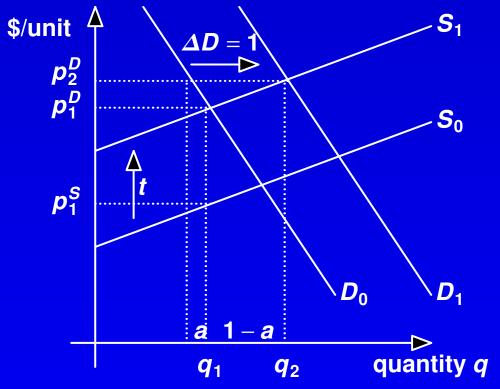




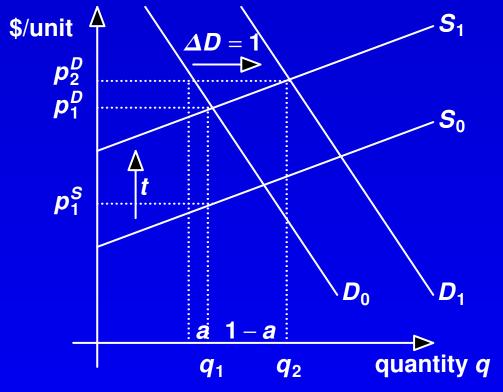




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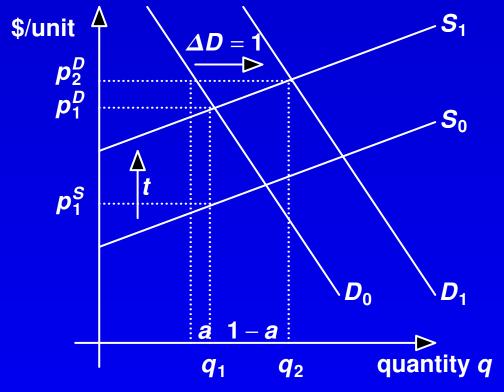


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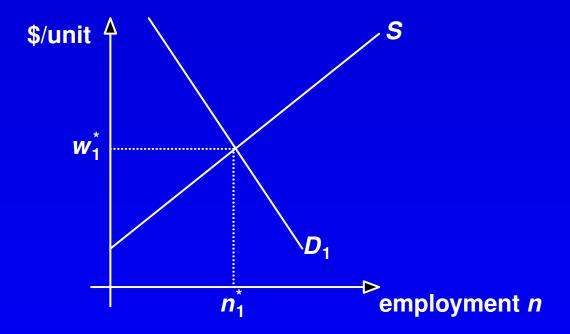
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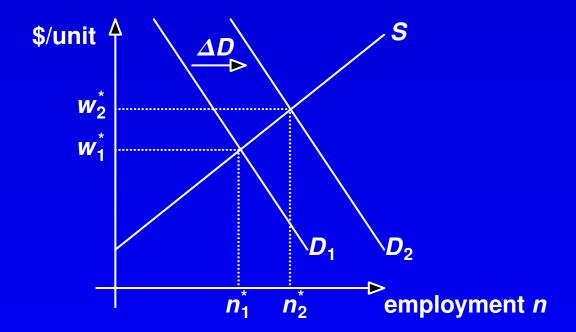


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- So $\Delta D \times p_S$ = the sum of the two areas

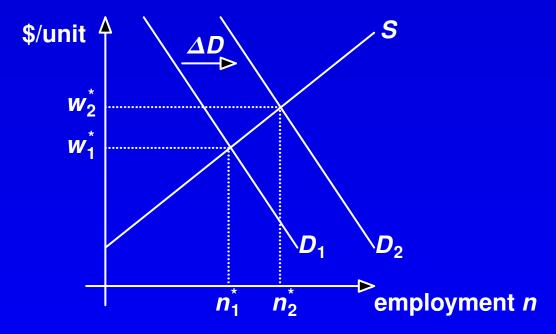
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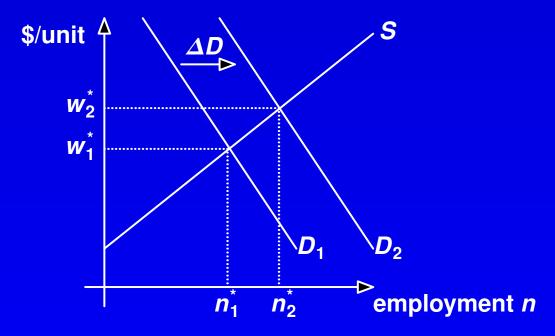
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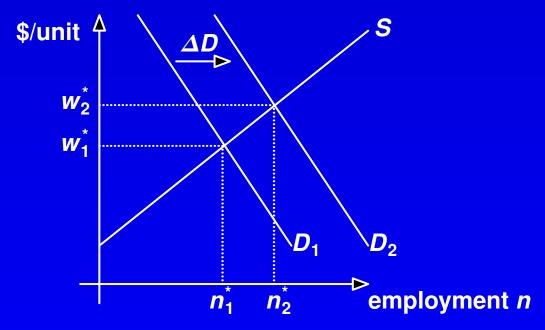
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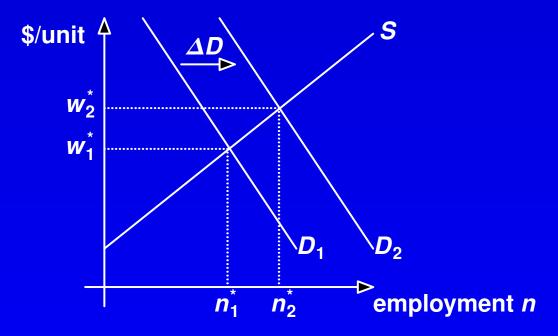
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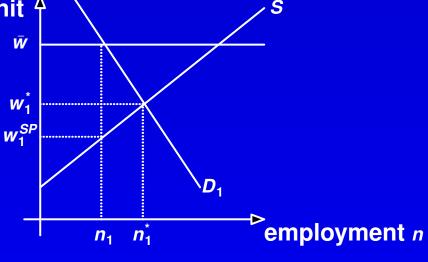
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- workers move from lower-paid to higher-paid jobs
- and there is no "involuntary" unemployment

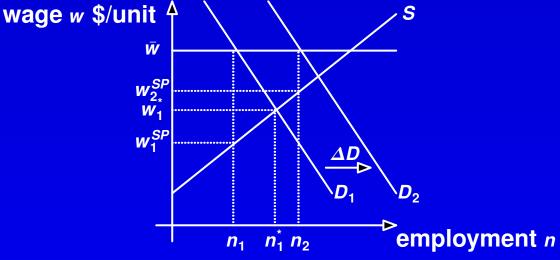
 D_1

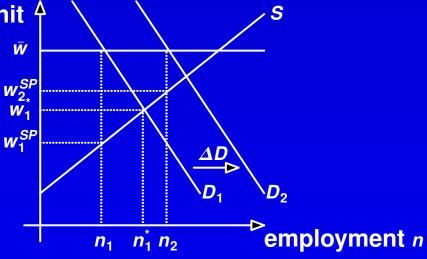
 n_1^*

employment n

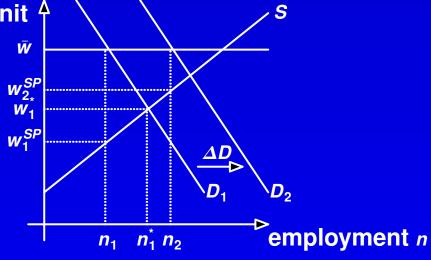
 w_1^*



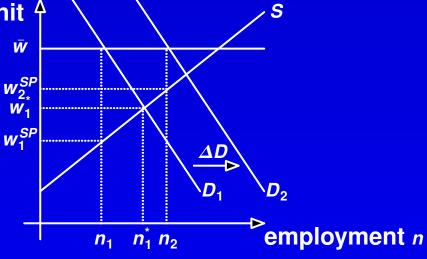




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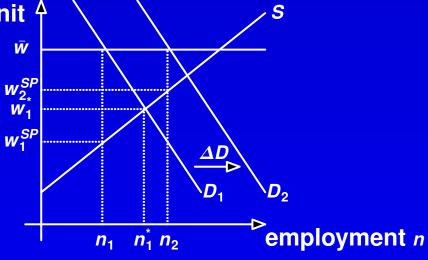


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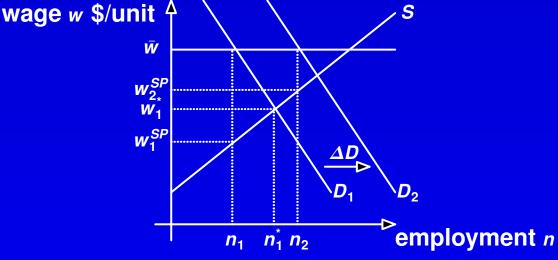


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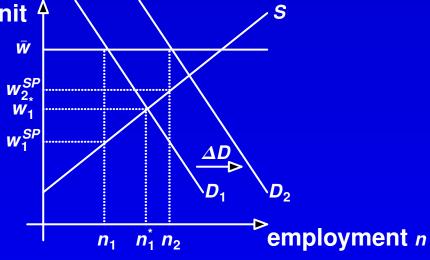
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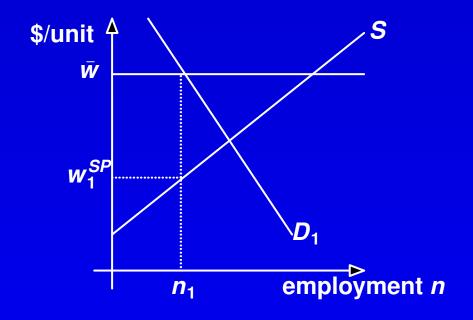
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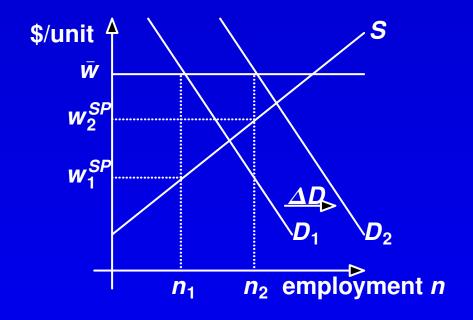


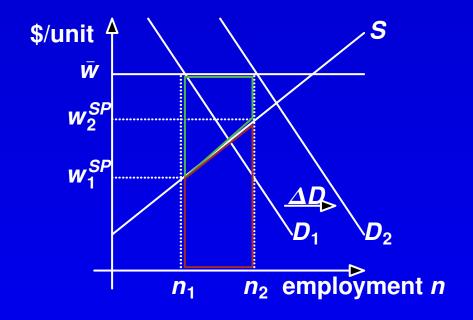
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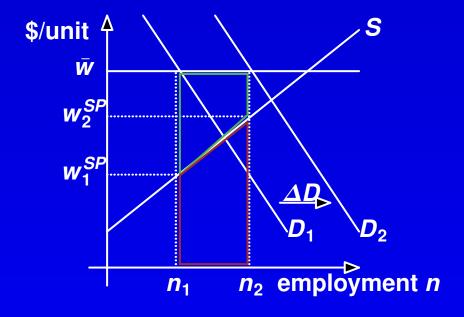


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 - Why? Because there is unemployment at minimum wage \bar{w} .

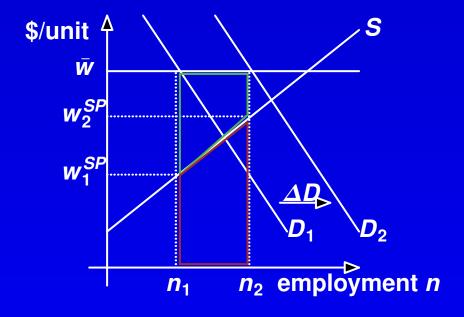








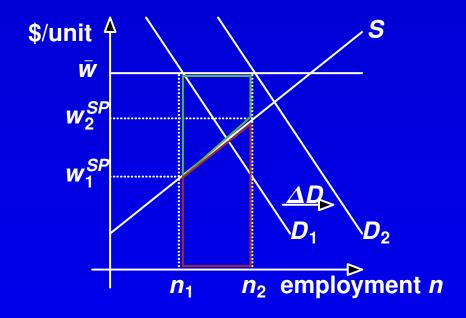
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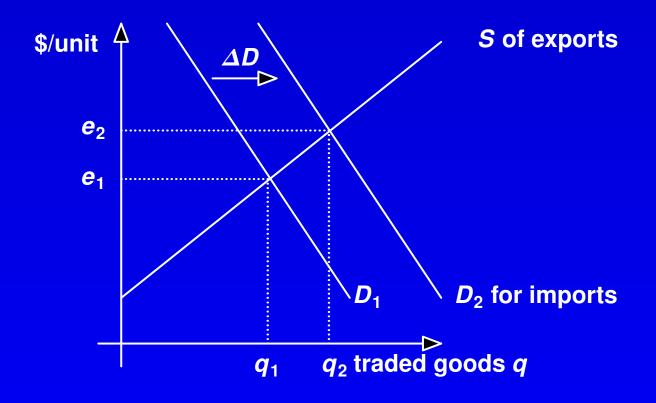
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- The change in the social cost = $\frac{w_1^{SP} + w_2^{SP}}{2} (n_2 n_1)$: brown area
- The opportunity cost of getting a job is less than \bar{w} , which is reflected in the supply curve. $w^{SP} = \frac{1}{2} (w_1^{SP} + w_2^{SP})$

1.5 Example 5: Foreign exchange [C&B Ch. 8; FP Ch. 9.2; DoF 3.8]



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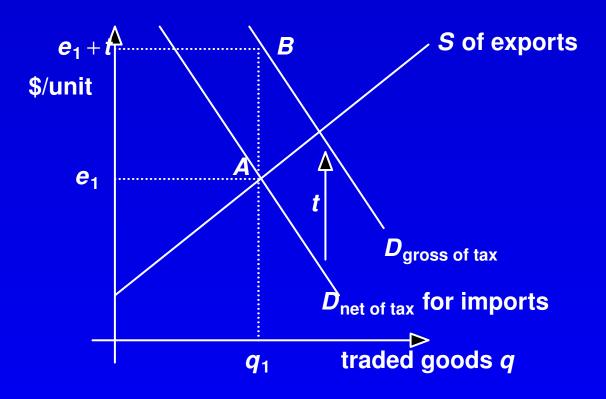
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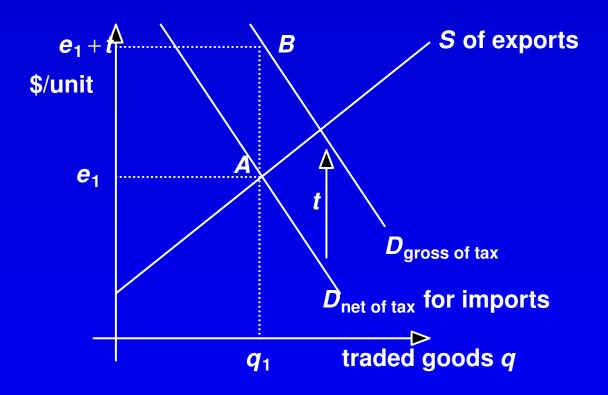
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- ∴ an upwards sloping supply curve S.



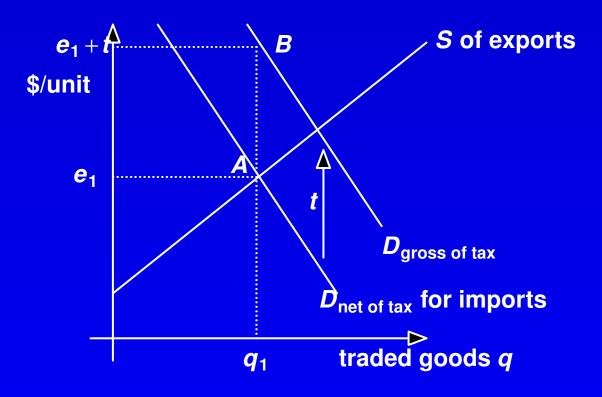
A tariff (tax) of t is now imposed on imports

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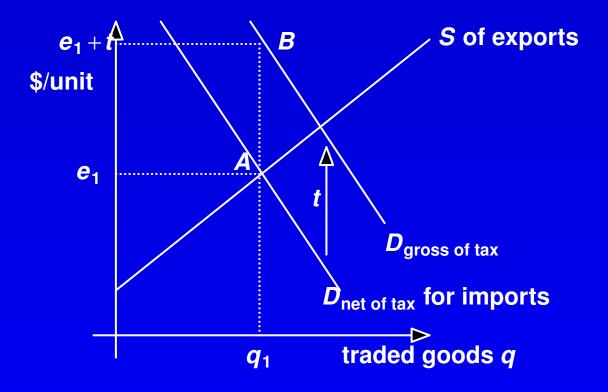
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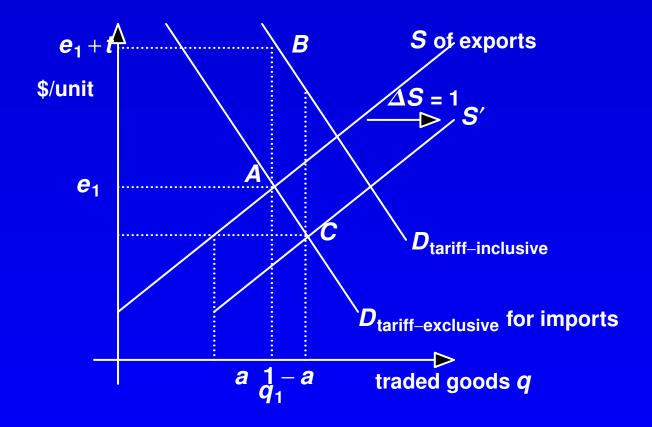
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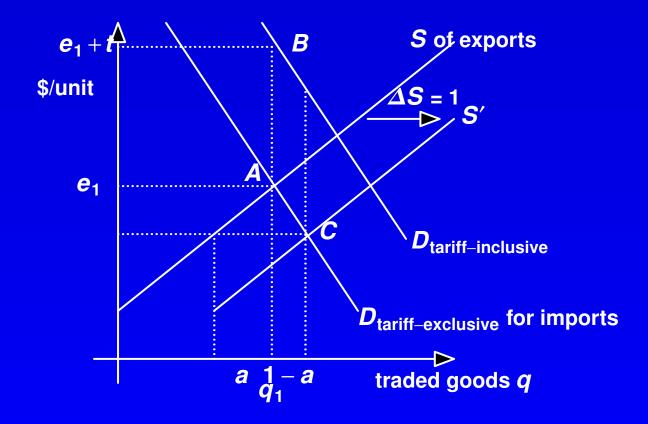
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- A represents the equilibrium value of exports
- B is the tariff-inclusive value of imports
- measured in terms of foreign currency, the value of imports and exports is equal.

Project now increases our supply of foreign exchange ΔS via additional exports. Let $\Delta S = 1$.



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this situation is similar to the tax example (p.5-12)

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- shadow exchange rate will exceed market exchange rate, since exports are under-valued by the market exchange rate. — The Gregory Thesis or Dutch Disease.

This lecture introduced the use of market prices — suitably adjusted to become *shadow prices* which accurately reflect the opportunity cost of the goods and services used by the project, whether produced in response to the project's demand (*incremental*) or bid away from existing uses (*displaced*) — in CBA studies.

Remember: No price change .. no displacement

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 How to adjust market prices for taxes (which are transfers, by and large).

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- How to adjust market prices for regulated prices, such as minimum wages with unemployment among the workers the project will hire. Shadow wages.