

Multi-Attribute Decision Making

Many decisions are based on other attributes than price. Choosing a car, for instance, although you might be looking in a particular price band. Comfort, performance, reliability, size, safety, style, image, equipment, handling, noise, running costs — these are some attributes of cars.

Example:

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Example: helping a family to buy a car

Steps:

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(1) *Clarify problem* keep an older car?
 use public transport?
 constraints? —

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-
- (1)
*Clarify
problem*
-
- keep an older car?
 - use public transport?
 - constraints? —
 - \$
 - manual transmission / auto?
 - size?
 - power steering?
 - ? 1. driving kids to school
 - ? 2. reliable & safe commuting vehicle?
 - ? 3. status symbol
 - ? 4. help on family holidays

Example (cont.):

Attributes: Price, handling & performance, overall safety, overall comfort, brakes, visibility, manufacturer's reputation (AFR 17/11/04)

| | | |
|-------------------|----------------------------|-------|
| _____ | (1) comfort 5A, or 1A + 5K | S_1 |
| (2) | (2) safe & reliable | S_2 |
| <i>Identify</i> | (3) status | S_3 |
| <i>objectives</i> | given the \$ constraint | |
| _____ | | |

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| _____ | | |

| | |
|-------------------------|--------------------------------|
| _____ | (1) + (3) subjective—judgement |
| (3) | intuition |
| <i>Measurement</i> | experience |
| <i>of effectiveness</i> | (2) less subjective |
| _____ | |

Additive Valuation

1.

Additive Valuation

1. Use scales for S_1, S_2, S_3
(1) (2) (3)

For each of the three attributes (1), (2), and (3), score the cars on a scale from 0 to 1.

- 2.

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The three weightings w_1, w_2, w_3 should be normalised:
 $\sum w_i = 1.$

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3. From part (1), each car j has a score for attribute i :
 $\therefore x_{ij}$ is the score of car j in attribute i .
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4. Choose the car with the highest total score, *or* iterate, until you feel happy with the scores, the weightings, and the final outcome.

Multiattribute Problem

CBA a subset

e.g. which bank ?

**quality of
service**

**interest
rates**

location

Comparing specific

outcomes

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- 3. Lexicographic ordering**
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- 4. Reducing search**
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- 6. Additive value models**

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but

1. Pairwise comparisons

“eye-balling”:

- OK for small number of attributes
- ? OK number of alternatives?
- large number of alternatives or attributes
- no complete preference ordering
- - but – time consuming, costly
 - continuous variables
 - no information for *delegation*

2. “Satisficing”



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→ iterative solution

if min levels too | *high*
low

So: useful, often used, attributes explicit

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- Examine the table on the next page, where countries' performances at the Atlanta Olympics are tabulated lexicographically.

This means there is no trade-off between numbers of Silver medals and numbers of Golds, so that Denmark (4 G, 1 S, 1 B) is ranked nineteenth, while Great Britain (1 G, 8 S, 5 B) is ranked thirty-sixth.

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- Or we could rank by total number of medals, which means equal trade-offs between Gold and Silver and Bronze.
- Or we could weight the medals, say, Gold = 3, Silver = 2, Bronze = 1, which still allows a trade-off, but not an equal trade-off.

Lexicographically Ranked by Gold, Silver, Bronze Medals (Atlanta)

| | <i>Gold</i> | <i>Silver</i> | <i>Bronze</i> | <i>Total</i> |
|----------------|-------------|---------------|---------------|--------------|
| United States | 44 | 32 | 25 | 101 |
| Russia | 26 | 21 | 16 | 63 |
| Germany | 20 | 18 | 27 | 65 |
| China | 16 | 22 | 12 | 50 |
| France | 15 | 7 | 15 | 37 |
| Italy | 13 | 10 | 12 | 35 |
| Australia | 9 | 9 | 23 | 41 |
| Cuba | 9 | 8 | 8 | 25 |
| Ukraine | 9 | 2 | 12 | 23 |
| South Korea | 7 | 15 | 5 | 27 |
| Poland | 7 | 5 | 5 | 17 |
| Hungary | 7 | 4 | 10 | 21 |
| Spain | 5 | 6 | 6 | 17 |
| Romania | 4 | 7 | 9 | 20 |
| Netherlands | 4 | 5 | 10 | 19 |
| Greece | 4 | 4 | 0 | 8 |
| Czech Republic | 4 | 3 | 4 | 11 |
| Switzerland | 4 | 3 | 0 | 7 |
| Denmark | 4 | 1 | 1 | 6 |
| Turkey | 4 | 1 | 1 | 6 |
| Canada | 3 | 11 | 8 | 22 |
| Bulgaria | 3 | 7 | 5 | 15 |
| Japan | 3 | 6 | 5 | 14 |
| Kazakhstan | 3 | 4 | 4 | 11 |
| Brazil | 3 | 3 | 9 | 15 |
| New Zealand | 3 | 2 | 1 | 6 |
| South Africa | 3 | 1 | 1 | 5 |
| Ireland | 3 | 0 | 1 | 4 |
| Sweden | 2 | 4 | 2 | 8 |
| Norway | 2 | 2 | 3 | 7 |
| Belgium | 2 | 2 | 2 | 6 |
| Nigeria | 2 | 1 | 3 | 6 |
| North Korea | 2 | 1 | 2 | 5 |
| Algeria | 2 | 0 | 1 | 3 |
| Ethiopia | 2 | 0 | 1 | 3 |
| Great Britain | 1 | 8 | 5 | 15 |
| Belarus | 1 | 6 | 8 | 15 |
| Kenya | 1 | 4 | 3 | 8 |
| Jamaica | 1 | 3 | 2 | 6 |
| Finland | 1 | 2 | 1 | 4 |
| Indonesia | 1 | 1 | 2 | 4 |
| Yugoslavia | 1 | 1 | 2 | 4 |
| Iran | 1 | 1 | 1 | 3 |
| Slovakia | 1 | 1 | 1 | 3 |

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e.g. which building to choose, given the two main uses for the building of Athletics and Crafts?

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| Building | Rank (ordinal) | |
|----------|----------------|--------|
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| B | 1 | 2 |
| C | 3 | 5 |
| D | 2 | 1 |
| E | 5 | 3 |

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So we see that:

D,B *dominate* C,A,E

B: 1,2 D: 2,1

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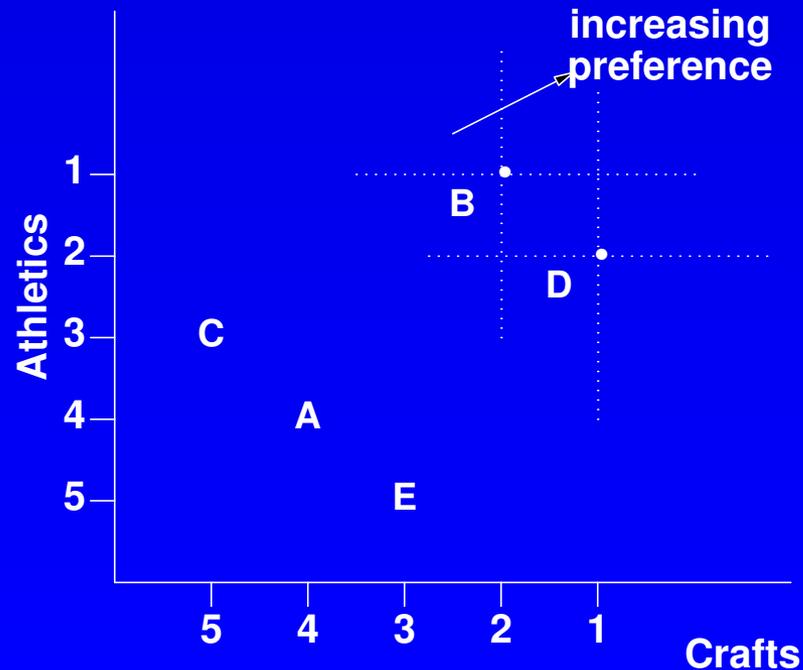
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[see the Hammond *HBR* reading in the Package.]

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e.g. which of five jobs to choose, given the five attributes of each job?

| Attributes / Characteristics | | | | | | |
|------------------------------|--------|--------------|--------------------|------------|-------|--|
| Job | Salary | Leisure Time | Working conditions | Co-workers | Where | |
| <i>A</i> | 2 | 3 | 3 | 2 | 2 | |
| <i>B</i> | 3 | 4 | 4 | 1 | 2 | |
| <i>C</i> | 3 | 3 | 2 | 3 | 3 | |
| <i>D</i> | 3 | 1 | 2 | 1 | 1 | |
| <i>E</i> | 1 | 2 | 1 | 2 | 2 | |

Freda has ranked the jobs in terms of each attribute.

E \mathcal{P} *A*
E \mathcal{P} *C*
D \mathcal{P} *B*

\therefore Freda's comparison is reduced to *D*, *E*

Even Swaps (cont.)

Spell out the measures of each attribute:

| Job | Salary | Leisure Time | Working conditions | Co-workers | Location |
|----------|--------|--------------|--------------------|------------|----------|
| <i>D</i> | \$90k | 8 days | W_D | C_D | L_D |
| <i>E</i> | \$100k | 5 days | W_E | C_E | L_E |

Q:

Even Swaps (cont.)

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Q: How much of \$100K would Freda be prepared to give up to get 3 additional leisure days/year?

A:

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Q: How much of \$100K would Freda be prepared to give up to get 3 additional leisure days/year?

A: \$25K → *E'*

| | | | | | |
|-----------|-----|---|-------|-------|-------|
| <i>D</i> | 90k | 8 | W_D | C_D | L_D |
| <i>E'</i> | 75k | 8 | W_E | C_E | L_E |

from above W_E (1st) > W_D (2nd)

Q:

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Q: How much of \$90k would Freda be prepared to give up to get W_E ?

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Q: How much of \$90k would Freda be prepared to give up to get W_E ?

A: \$10k → *D'*

“pricing out”

Even Swaps (cont.)

| | | | | | |
|-------|---------|---|-------|-------|-------|
| D' | \$80k | 8 | W_E | C_D | L_D |
| E' | \$75k | 8 | W_E | C_E | L_E |
| D' | \$80k | 8 | W_E | C_D | L_D |
| E'' | \$70k | 8 | W_E | C_D | L_E |
| D'' | \$72.5k | 8 | W_E | C_D | L_E |
| E'' | \$70k | 8 | W_E | C_D | L_E |

i.e. all attributes “priced out” by Freda, whose choice is job D

$D' \succ D'' - ?$

$E' \succ B'' - ?$

$D \succ D' - ?$

$E \succ B' - ?$

$E'' \succ D''$

$\therefore E \succ D$

$D \succ D'' \succ E'' \succ E \Rightarrow D \succ E$

6. Additive Value Models

e.g.

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e.g. three projects: A, B, & C

three attributes:

| | | | |
|--------------------|-----------|---|----------------------|
| Net Present Value | <i>PV</i> | ⊕ | the more, the better |
| Time to Completion | <i>T</i> | ⊖ | the less, the better |
| Impact | <i>I</i> | ⊕ | |

| | A | B | C |
|-----|-------|-------|-------|
| NPV | \$20m | \$15m | \$25m |
| T | 8y | 5y | 12y |
| I | 200k | 300k | 100K |

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| Time to Completion | T | \ominus | the less, the better |
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○ Independence ○

If the trade-off between $\{PV \ \& \ T\}$ is independent of the level of I

& if the trade off between $\{T, I\}$ is independent of the level of PV

then $\{PV \ \& \ I\}$ are independent of T .

i.e. *Preference Independence* of PV, T, I

Value Function

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 - where $v_{ij}(\cdot)$ is a “relative value preference of attribute i for project j ”
 $v_{ij} \in [0, 1]$
 - where w_i are attribute weights, $\sum w_i = 1$
- Project $j \rightarrow$ score V_j & can compare projects : V_j to obtain ranking

| e.g. | w_i | A $j=1$ | v_{i1} | B $j=2$ | v_{i2} | C $j=3$ | v_{i3} |
|------|-------|------------|----------|------------|----------|------------|----------|
| NPV | 0.9 | \$20m | 0.5 | \$15m | 0 | \$25m | 1 |
| T | 0.06 | 8y | 0.6 | 5y | 1 | 12y | 0 (-ve) |
| I | 0.04 | 200k | 0.8 | 300k | 1 | 100k | 0 |

e.g. x_{23} = level of attribute T in Project 3 = 12.

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Value Function

$$V(\text{project } j) = \sum_i^{\text{attributes}} w_i [v_{ij}(x_{ij})]$$

- where x_{ij} is the level of attribute i in project j
 - where $v_{ij}(\cdot)$ is a “relative value preference of attribute i for project j ”
 $v_{ij} \in [0, 1]$
 - where w_i are attribute weights, $\sum w_i = 1$
- Project $j \rightarrow$ score V_j & can compare projects : V_j to obtain ranking

| e.g. | w_i | A $j=1$ | v_{i1} | B $j=2$ | v_{i2} | C $j=3$ | v_{i3} |
|------|-------|------------|----------|------------|----------|------------|----------|
| NPV | 0.9 | \$20m | 0.5 | \$15m | 0 | \$25m | 1 |
| T | 0.06 | 8y | 0.6 | 5y | 1 | 12y | 0 (-ve) |
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project A: $V_A = 0.9 \times 0.5 + 0.06 \times 0.6 + 0.04 \times 0.8 = 0.518$

$V_B = 0.9 \times 0 + 0.06 \times 1 + 0.04 \times 1 = 0.1$

| | A l t e r n a t i v e s | | | | |
|------------------------|--------------------------------------|-----------------------|------------------------------|---------------|-----------------------|
| | Job A | Job B | Job C | Job D | Job E |
| Objectives | | | | | |
| Weekly salary | \$2000 | \$2400 | \$1800 | \$1900 | \$2200 |
| Flexibility | mod | low | high | mod | none |
| Business skills | computer | people man. | operations | org. | time man. |
| Development | | computer | computer | | multitasking |
| Annual leave | 14 | 12 | 10 | 15 | 12 |
| Benefits | health, dental retirement | health, dental | health retirement | health | health, dental |
| Employment | great | good | good | great | boring |
| Location | Syd | Melb | Syd | Bris | Perth |

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5. Don't double count.

Only individuals matter

+

All individuals matter equally
(or: a \$ is a \$, no matter whose)

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(See Dixit & Pindyck and Bruun & Bason)

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7. **the Winner's Curse when choosing one of several: the estimates of future costs and benefits are not unbiased in the most attractive project (highest benefits – costs): possibility of negative NPV.**

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or follow-on investments, with distant and uncertain payoffs. Often, learning more about future options is most valuable.

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Answer: the principles of risk-neutral valuation with the Black-Scholes option pricing techniques.