Multi-Attribute Decision Making

Many decisions are based on other attributes than price. Choosing a car, for instance, although you might be looking in a particular price band. Comfort, performance, reliability, size, safety, style, image, equipment, handling, noise, running costs — these are some attributes of cars.

Example: **helping a family to buy a car**

Steps: **(1) Clarify problem; (2) Identify objectives; (3) Measurement of effectiveness.**

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(1) *Clarify problem*

keep an older car?
use public transport?
constraints? —
$
manual transmission / auto?
size?
power steering?
? 1. driving kids to school
? 2. reliable & safe commuting vehicle?
? 3. status symbol
? 4. help on family holidays

Example (cont.):

Attributes: Price, handling & performance, overall safety, overall comfort, brakes, visibility, manufacturer's reputation (AFR 17/11/04)

(2) *Identify objectives*	**(1) comfort 5A, or 1A + 5K**	S_1
	(2) safe & reliable	S_2
	(3) status	S_3
	given the $ constraint	

(3) *Measurement of effectiveness*	**(1) + (3) subjective—judgement** **intuition** **experience**
	(2) less subjective

Additive Valuation

1. **Use scales for S_1, S_2, S_3**
   (1) (2) (3)

   For each of the three attributes (1), (2), and (3), score the cars on a scale from 0 to 1.

2. **Subject to the $ constraint, now weight the three attributes: i.e.**
   - **How important is the first attribute (comfort) in the total decision? $\rightarrow w_1$**
   - **How important the second (safety and reliability)? $\rightarrow w_2$**
   - **The third (status)? $\rightarrow w_3$**

   The three weightings w_1, w_2, w_3 should be normalised: $\sum w_i = 1$.

3. **From part (1), each car j has a score for attribute i:**

   $\therefore$ x_{ij} is the score of car j in attribute i.

   $\therefore$ Each car's total score can be calculated: $\sum_i x_{ij} w_i \rightarrow$ score for car j

4. **Choose the car with the highest total score, *or* iterate, until you feel happy with the scores, the weightings, and the final outcome.**

Multiattribute Problem

CBA a subset

e.g. which bank ?

quality of service	interest rates	location

Comparing specific | ***outcomes***

~~***projects***~~

There are six ways: (Perry & Dillon in the Package)

1. **Pairwise comparisons**
2. **“Satisficing”**
3. **Lexicographic ordering**
4. **Reducing search**
5. **Even swaps, or Pricing out**
6. **Additive value models**

1. Pairwise comparisons

"eye-balling":

- **OK for small number of attributes**
- **? OK number of alternatives?**
- **large number of alternatives or attributes**
- **no complete preference ordering**
-

but – time consuming, costly

– continuous variables

→ no information for *delegation*

2. “Satisficing”

- **set minimum levels (“satisfy”) of all attributes but one (the “target” attribute)**
- **choose the project/outcome/action with the highest level of the target**

→ **iterative solution**

if min levels too | *high* / *low*

So: *useful, often used, attributes explicit*

3. Lexicographic Ordering

How to:

- **rank attributes;**
- **choose project with the highest Attribute 1;**
- **only consider Attribute 2 if there is a tie in terms of Attribute 1.**
- **Using the letters of the alphabet in order, this is how dictionaries (or lexicons) order words — hence, lexicographic.**
- **Examine the table on the next page, where countries' performances at the Atlanta Olympics are tabulated lexicographically.**

  This means there is no trade-off between numbers of Silver medals and numbers of Golds, so that Denmark (4 G, 1 S, 1 B) is ranked nineteenth, while Great Britain (1 G, 8 S, 5 B) is ranked thirty-sixth.
- **Or we could rank by total number of medals, which means equal trade-offs between Gold and Silver and Bronze.**
- **Or we could weight the medals, say, Gold = 3, Silver = 2, Bronze = 1, which still allows a trade-off, but not an equal trade-off.**

Lexicographically Ranked by Gold, Silver, Bronze Medals (Atlanta)

	Gold	*Silver*	*Bronze*	*Total*
United States	44	32	25	101
Russia	26	21	16	63
Germany	20	18	27	65
China	16	22	12	50
France	15	7	15	37
Italy	13	10	12	35
Australia	9	9	23	41
Cuba	9	8	8	25
Ukraine	9	2	12	23
South Korea	7	15	5	27
Poland	7	5	5	17
Hungary	7	4	10	21
Spain	5	6	6	17
Romania	4	7	9	20
Netherlands	4	5	10	19
Greece	4	4	0	8
Czech Republic	4	3	4	11
Switzerland	4	3	0	7
Denmark	4	1	1	6
Turkey	4	1	1	6
Canada	3	11	8	22
Bulgaria	3	7	5	15
Japan	3	6	5	14
Kazakhstan	3	4	4	11
Brazil	3	3	9	15
New Zealand	3	2	1	6
South Africa	3	1	1	5
Ireland	3	0	1	4
Sweden	2	4	2	8
Norway	2	2	3	7
Belgium	2	2	2	6
Nigeria	2	1	3	6
North Korea	2	1	2	5
Algeria	2	0	1	3
Ethiopia	2	0	1	3
Great Britain	1	8	5	15
Belarus	1	6	8	15
Kenya	1	4	3	8
Jamaica	1	3	2	6
Finland	1	2	1	4
Indonesia	1	1	2	4
Yugoslavia	1	1	2	4
Iran	1	1	1	3
Slovakia	1	1	1	3

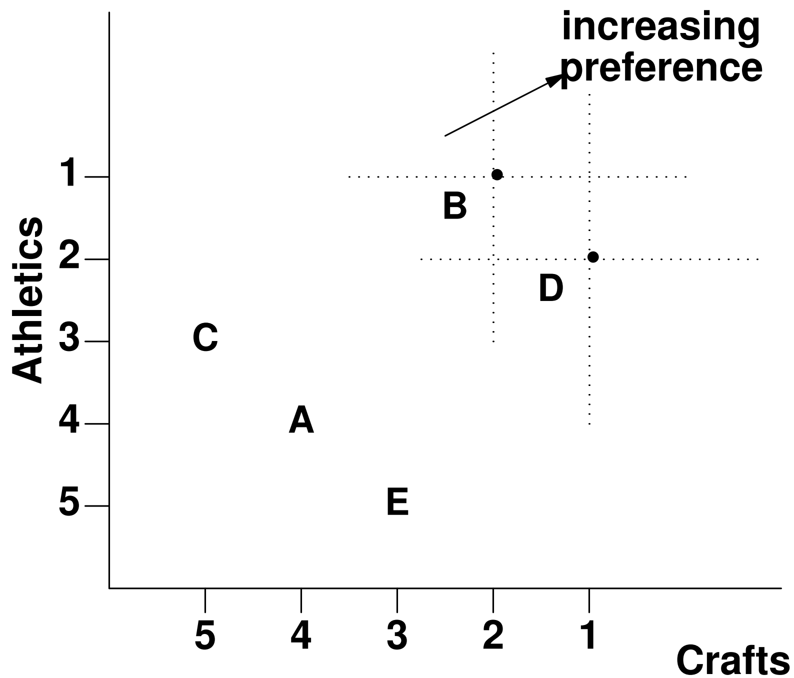
4. Reducing Search

e.g. which building to choose, given the two main uses for the building of Athletics and Crafts?

	Rank (ordinal)	
Building	**Athletics**	**Crafts**
A	4	4
B	1	2
C	3	5
D	2	1
E	5	3

So we see that:

D,B ***dominate*** **C,A,E**
B: 1,2   D: 2,1



5. Even Swaps, or Pricing Out

[see the Hammond *HBR* reading in the Package.]

e.g. which of five jobs to choose, given the five attributes of each job?

Attributes / Characteristics

Job	Salary	Leisure Time	Working conditions	Co-workers	Where
A	2	3	3	2	2
B	3	4	4	1	2
C	3	3	2	3	3
D	3	1	2	1	1
E	1	2	1	2	2

Freda has ranked the jobs in terms of each attribute.

$E \; \mathcal{P} \; A$
$E \; \mathcal{P} \; C$
$D \; \mathcal{P} \; B$

$\therefore$ **Freda's comparison is reduced to *D*, *E***

Even Swaps (cont.)

Spell out the measures of each attribute:

Job	Salary	Leisure Time	Working conditions	Co-workers	Location
D	\$90k	8 days	W_D	C_D	L_D
E	\$100k	5 days	W_E	C_E	L_E

Q: How much of \$100K would Freda be prepared to give up to get 3 additional leisure days/year?

A: \$25K → *E*′

D	90k	8	W_D	C_D	L_D
E′	75k	8	W_E	C_E	L_E

from above W_E (1st) > W_D (2nd)

Q: How much of \$90k would Freda be prepared to give up to get W_E?

A: \$10k → D′ "***pricing out***"

Even Swaps (cont.)

D'	\$80k	8	W_E	C_D	L_D
E'	\$75k	8	W_E	C_E	L_E
D'	\$80k	8	W_E	C_D	L_D
E''	\$70k	8	W_E	C_D	L_E
D''	\$72.5k	8	W_E	C_D	L_E
E''	\$70k	8	W_E	C_D	L_E

i.e. all attributes "priced out" by Freda, whose choice is job *D*

$$D' \mathcal{I} D'' - ?$$
$$E' \mathcal{I} B'' - ?$$
$$D \mathcal{I} D' - ?$$
$$E \mathcal{I} B' - ?$$
$$E'' \mathcal{I} D''$$
$$\therefore \; E \mathcal{I} D$$

$$D \mathcal{I} D'' \mathcal{P} E'' \mathcal{I} E \Rightarrow D \mathcal{P} E$$

6. Additive Value Models

e.g. three projects: A, B, & C

three attributes:

Net Present Value	***PV***	⊕	**the more, the better**
Time to Completion	***T***	⊖	**the less, the better**
Impact	***I***	⊕	

	A	B	C
NPV	**$20m**	**$15m**	**$25m**
T	**8y**	**5y**	**12y**
I	**200k**	**300k**	**100K**

○ ***Independence*** ○

If the trade-off between {*PV* & *T*} is independent of the level of *I*

& if the trade off between {*T*, *I*} is independent of the level of *PV*

***then* {*PV* & *I*} are independent of *T*.**

i.e. *Preference Independence* of *PV*, *T*, *I*

Value Function

$$V(\text{project } j) = \sum_{i}^{\text{attributes}} w_i[v_{ij}(x_{ij})]$$

- where x_{ij} is the level of attribute i in project j
- where $v_{ij}(.)$ is a "relative value preference of attribute i for project j"
  $v_{ij} \in [0, 1]$
- where w_i are attribute weights, $\sum w_i = 1$
  Project $j \rightarrow$ score V_j & can compare projects : V_j to obtain ranking

e.g.	w_i	A j=1	v_{i1}	B j=2	v_{i2}	C j=3	v_{i3}
NPV	0.9	\$20m	0.5	\$15m	0	\$25m	1
T	0.06	8y	0.6	5y	1	12y	0 (–ve)
I	0.04	200k	0.8	300k	1	100k	0

e.g. x_{23} = level of attribute T in Project 3 = 12.

$\sum w_i = 1,\ w_i \geq 0$ attribute weights

project A: $V_A = 0.9 \times 0.5 + 0.06 \times 0.6 + 0.04 \times 0.8 = 0.518$

$V_B = 0.9 \times 0 + 0.06 + 0.04 = 0.1$

	Alternatives				
	Job A	Job B	Job C	Job D	Job E
Objectives					
Weekly salary	**$2000**	**$2400**	**$1800**	**$1900**	**$2200**
Flexibility	**mod**	**low**	**high**	**mod**	**none**
Business skills	**computer**	**people man.**	**operations**	**org.**	**time man.**
Development		**computer**	**computer**		**multitasking**
Annual leave	**14**	**12**	**10**	**15**	**12**
Benefits	**health, dental retirement**	**health, dental**	**health retirement**	**health**	**health, dental**
Employment	**great**	**good**	**good**	**great**	**boring**
Location	**Syd**	**Melb**	**Syd**	**Bris**	**Perth**

Landsburg

1. **Tax revenues are not a net benefits (when looking from society's viewpoint) and a reduction in tax revenues is not a net cost.**
2. **A cost is a cost, no matter who bears it.**
3. **A good is a good, no matter who owns it.**
4. **Voluntary consumption is a good thing.**
5. **Don't double count.**

Only individuals matter
+
All individuals matter equally
(or: a $ is a $, no matter whose)

Real Options

(See Dixit & Pindyck and Bruun & Bason)

Disadvantages of NPV/DCF (especially for private firms):

1. **positive-NPV opportunities might be bid away as firms enter (strategic rivalry)**
2. **allocation of overhead costs in a multi-project setting is non-trivial**
3. **assumption of reinvestment at the entire project's rate is questionable**
4. **the risk adjustment (β) of the discount rate depends on: project life, growth trend in the expected DCF, etc.**
5. **interdependencies among projects: spillovers, asymmetric (skewed) outcomes, etc.**
6. **investments are sunk (sometimes assumed not)**
7. **the Winner's Curse when choosing one of several:**
   the estimates of future costs and benefits are not unbiassed in the most attractive project (highest benefits – costs): possibility of negative NPV.

What if there are options present:

- — timing: wait
- — operational: flexibility & discretion once underway
- — growth: future options contingent on this project

Then NPV/DCF:

1. ***with timing options:***
   if projects are exclusive or investment budgets limited, then projects effectively compete with themselves over time.
2. ***with operational options:***
   including
   - — temporary shutdowns
   - — expanding or scaling down operations
   - — switching between inputs, outputs, or processes

   Can create value, but skew the return distribution: must use options techniques.
3. ***with growth options:***
   or follow-on investments, with distant and uncertain payoffs. Often, learning more about future options is most valuable.

Why not use Decision Analysis?

***Plus*: a Decision Tree does model asymmetries and paths, but**

***Minus*: as the value of the underlying asset (the project) changes over time, so does its risk and so the correct risk premium.**

***Answer*: the principles of risk-neutral valuation with the Black-Scholes option pricing techniques.**