Tests of the theory of sequential-move games seem to suggest that actual play shows the irrationality of the players or the failure of the theory to adequately predict behavior. The counterargument points out the complexity of actual preferences for different possible outcomes and the usefulness of strategic theory for identifying optimal actions when actual preferences are known.

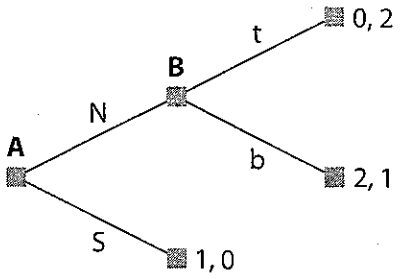
KEY TERMS

action node (46)
backward induction (54)
branch (46)
decision node (46)
decision tree (46)
equilibrium path of play (58)
extensive form (46)
first-mover advantage (60)
game tree (46)
initial node (46)
intermediate valuation function (65)
move (48)
node (46)
path of play (58)
prune (52)
rollback (54)
rollback equilibrium (54)
root (46)
second-mover advantage (59)
terminal node (48)

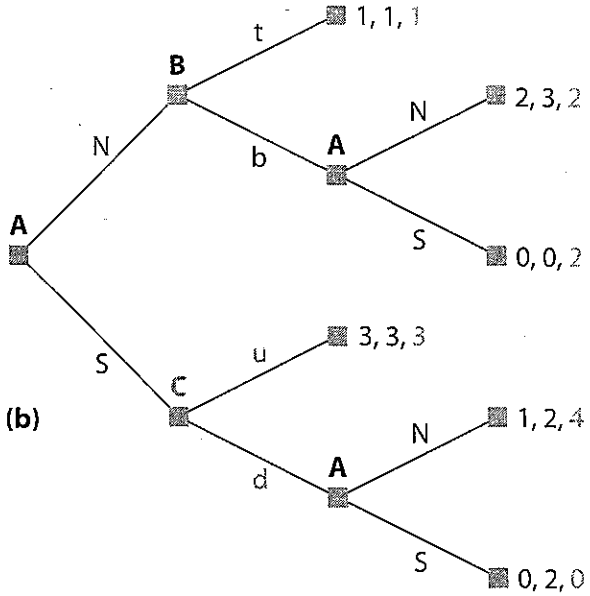
EXERCISES

1. Suppose two players, First and Second, take part in a sequential-move game. First moves first, Second moves second, and each player moves only once.
   (a) Draw a game tree for a game in which First has two possible actions (Up or Down) at each node and Second has three possible actions (Top, Middle, or Bottom) at each node. How many of each node type—decision and terminal—are there?
   (b) Draw a game tree for a game in which First and Second each have three possible actions (Sit, Stand, or Jump) at each node. How many of the two node types are there?
   (c) Draw a game tree for a game in which First has four possible actions (North, South, East, or West) at each node and Second has two possible actions (Stay or Go) at each node. How many of the two node types are there?

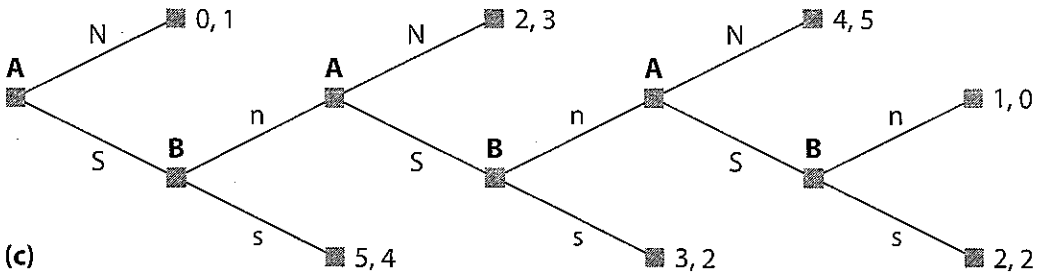
2. Use rollback to find equilibria for the following games:



(a)



(b)



(c)

In each case, how many pure strategies (complete plans of action) are available to each player? For each game, write out all of the pure strategies for each player.

3. For each of the games illustrated in Exercise 2, identify the equilibrium outcome and the complete equilibrium strategy for each player.

4. "In a sequential-move game, the player who moves first is sure to win." Is this statement true or false? State the reason for your answer in a few brief sentences, and give an example of a game that illustrates your answer.

5. Consider the rivalry between Airbus and Boeing to develop a new commercial jet aircraft. Suppose Boeing is ahead in the development process and Airbus is considering whether to enter the competition. If Airbus stays out, it earns zero profit, whereas Boeing enjoys a monopoly and earns a profit of $1 billion. If Airbus decides to enter and develop the rival airplane, then Boeing has to decide whether to accommodate Airbus peaceably or to wage a price war. In the event of peaceful competition, each firm will make a

profit of $300 million. If there is a price war, each will lose $100 million because the prices of airplanes will fall so low that neither firm will be able to recoup its development costs.

Draw the tree for this game. Find the rollback equilibrium and describe the firms' equilibrium strategies.

6. The centipede game illustrated in Figure 3.8 can be solved by using rollback without drawing the complete game tree. In the version of the game discussed in the text, players A and B alternately had the opportunity to claim or pass a growing pile of dimes (to a maximum of 10) placed on the table. Suppose the rules of the games were changed so that:
   (a) Player A gets a nickel reward every time she passes, giving her opponent another turn. Find the rollback equilibrium strategies for each player.
   (b) Two rounds of the game are played with the same two players, A and B. In the first round, A may not keep more than five dimes, and B may not keep more than nine. Find the rollback equilibrium strategies for each player.
   (c) Two rounds of the game are played with the same two players, A and B. In the first round, A may not keep more than five dimes, and B may not keep more than four. Find the rollback equilibrium strategies for each player.

7. Two players, Amy and Beth, take turns choosing numbers; Amy goes first. On her turn, a player may choose any number between 1 and 10, inclusive, and this number is added to a running total. When the running total of both players' choices reaches 100, the game ends. Consider two alternative endings: (i) the player whose choice of number takes the total to exactly 100 is the winner and (ii) the player whose choice of number causes this total to equal or exceed 100 is the loser. For each case, answer the following questions:
   (a) Who will win the game?
   (b) What are the optimal strategies (complete plans of action) for each player?

8. Consider three major department stores—Big Giant, Titan, and Frieda's—contemplating opening a branch in one of two new Boston-area shopping malls. Urban Mall is located close to the large and rich population center of the area; it is relatively small and can accommodate at most two department stores as "anchors" for the mall. Rural Mall is farther out in a rural and relatively poorer area; it can accommodate as many as three anchor stores. None of the three stores wants to have its store in both malls because there is sufficient overlap of customers between the malls that locating in both would just be competing with oneself. Each store prefers to be in a mall with one or more other department stores than to be alone in the same mall, because a mall with multiple department stores will attract sufficiently many more total customers that each store's profit will be higher. Further, each store prefers Urban Mall to Rural Mall because of the richer customer base. Each store

must choose between trying to get a space in Urban Mall (knowing that, if the attempt fails, they will try for a space in Rural Mall) and trying to get a space in Rural Mall directly (without even attempting to get into Urban Mall).

In this case, the stores rank the five possible outcomes as follows: 5 (best), in Urban Mall with one other department store; 4, in Rural Mall with one or two other department stores;. 3, alone in Urban Mall; 2, alone in Rural Mall; and 1 (worst), alone in Rural Mall after having attempted to get into Urban Mall and failed, by which time other nondepartment stores have signed up the best anchor locations in Rural Mall.

The three stores are sufficiently different in their managerial structures that they experience different lags in doing the paperwork required to request an expansion space in a new mall. Frieda's moves quickly, followed by Big Giant, and finally by Titan, which is the least efficient in readying a location plan. When all three have made their requests, the malls decide which stores to let in. Because of the name recognition that both Big Giant and Titan have with the potential customers, a mall would take either (or both) of those stores before it took Frieda's. Thus, Frieda's does not get one of the two spaces in Urban Mall if all three stores request those spaces; this is true even though Frieda's moves first.

(a) Draw the game tree for this mall location game.

(b) Illustrate the rollback pruning process on your game tree and use the pruned tree to find the rollback equilibrium. Describe the equilibrium by using the (complete) strategies employed by each department store. What are the payoffs to each store at the rollback equilibrium outcome?

9. There are two distinct proposals, A and B, being debated in Washington. The Congress likes proposal A, and the president likes proposal B. The proposals are not mutually exclusive; either or both or neither may become law. Thus there are four possible outcomes, and the rankings of the two sides are as follows, where a larger number represents a more favored outcome.

Outcome	Congress	President
A becomes law	4	1
B becomes law	1	4
Both A and B become law	3	3
Neither (status quo prevails)	2	2

(a) The moves in the game are as follows. First, the Congress decides whether to pass a bill and whether it is to contain A or B or both. Then the president decides whether to sign or veto the bill. Congress does not have enough votes to override a veto. Draw a tree for this game and find the rollback equilibrium.

(b) Now suppose the rules of the game are changed in only one respect: the president is given the extra power of a line-item veto. Thus, if the Congress passes a bill containing both A and B, the president may choose not only to sign or veto the bill as a whole, but also to veto just one of the two items. Show the new tree and find the rollback equilibrium.

(c) Explain intuitively why the difference between the two equilibria arises.

10. Consider the Survivor game tree illustrated in Figure 3.9. Suppose that, unlike in Figure 3.9, you want to use only general values for the various probabilities. In particular, suppose that the probability of winning the immunity challenge when Rich chooses Continue is x for Rich, y for Kelly, and $1 - x - y$ for Rudy; similarly, the probability of winning when Rich chooses Give Up is z for Kelly and $1 - z$ for Rudy. Further, suppose that Rich's chance of being picked by the jury is p if he has won immunity and has voted off Rudy; his chance of being picked is q if Kelly has won immunity and has voted off Rudy. Continue to assume that, if Rudy wins immunity, he keeps Rich with probability 1, and that Rudy wins the game with probability 1 if he ends up in the final two.

(a) What is the algebraic formula for the probability, in terms of p, q, x, and y, that Rich wins the million dollars if he chooses Continue? What is the probability that he wins if he chooses Give Up? Can you determine Rich's optimal strategy with only this level of information?

(b) The discussion in Section 3.7 suggests that Give Up is optimal for Rich as long as (i) Kelly is very likely to win the immunity challenge once Rich gives up and (ii) Rich wins the jury's final vote more often when Kelly has voted out Rudy than when Rich has done so. Write out expressions entailing the general probabilities (p, q, x, y) that summarize these two conditions.

(c) Suppose that the two conditions from part **b** hold. Prove that Rich's optimal strategy is Give Up.