requires players to achieve coordination by some means. If a game has no equilibrium in *pure strategies,* we must look for an equilibrium in *mixed strategies,* the analysis of which is presented in Chapters 7 and 8.

KEY TERMS

assurance game (107)
battle of the sexes (108)
belief (89)
best response (87)
best-response analysis (99)
cell-by-cell inspection (89)
chicken (109)
constant-sum game (85)
convergence of expectations (107)
coordination game (105)
dominance solvable (96)
dominant strategy (92)
dominated strategy (92)
enumeration (89)
focal point (106)
game matrix (84)
game table (84)
iterated elimination of dominated strategies (96)
maximin (100)
minimax (100)
minimax method (99)
mixed strategy (84)
Nash equilibrium (87)
normal form (84)
payoff table (84)
prisoners' dilemma (90)
pure coordination game (105)
pure strategy (84)
strategic form (84)
successive elimination of dominated strategies (96)
zero-sum game (85)

EXERCISES

1. "If a player has a dominant strategy in a simultaneous-move game, then she is sure to get her best possible outcome." True or false? Explain and give an example of a game that illustrates your answer.

2. Find all Nash equilibria in pure strategies for the zero-sum games in the following tables by checking for dominant strategies and using iterated dominance. Verify your answers by using the minimax method.

(a)

		COLUMN	
		Left	Right
ROW	Up	1	4
	Down	2	3

(b)

		COLUMN	
		Left	Right
ROW	Up	1	2
	Down	4	3

(c)

		COLUMN		
		Left	Middle	Right
ROW	Up	5	3	1
	Straight	6	2	1
	Down	1	0	0

(d)

		COLUMN		
		Left	Middle	Right
ROW	Up	5	3	2
	Straight	6	4	3
	Down	1	6	0

3. Find all Nash equilibria in pure strategies in the following non-zero-sum games. Describe the steps that you used in finding the equilibria.

(a)

		COLUMN	
		Left	Right
ROW	Up	2, 4	1, 0
	Down	6, 5	4, 2

(b)

		COLUMN	
		Left	Right
ROW	Up	1, 1	0, 1
	Down	1, 0	1, 1

(c)

		COLUMN		
		Left	Middle	Right
ROW	Up	0, 1	9, 0	2, 3
	Straight	5, 9	7, 3	1, 7
	Down	7, 5	10, 10	3, 5

(d)

		COLUMN		
		West	Center	East
ROW	North	2, 3	8, 2	10, 6
	Up	3, 0	4, 5	6, 4
	Down	5, 4	6, 1	2, 5
	South	4, 5	2, 3	5, 2

4. Check the following game for dominance solvability. Find a Nash equilibrium.

		COLUMN		
		Left	Middle	Right
ROW	Up	4, 3	2, 7	0, 4
	Down	5, 5	5, −1	−4, −2

5. Find all of the pure-strategy Nash equilibria for the following game. Describe the process that you used to find the equilibria. Use this game to ex-

plain why it is important to describe an equilibrium by using the strategies employed by the players, not merely by the payoffs received in equilibrium.

		COLUMN		
		Left	Center	Right
ROW	Up	1, 2	2, 1	1, 0
	Level	0, 5	1, 2	7, 4
	Down	−1, 1	3, 0	5, 2

6. The game known as the *battle of the Bismarck Sea* (named for that part of the southwestern Pacific Ocean separating the Bismarck Archipelago from Papua-New Guinea) summarizes a well-known game actually played in a naval engagement between the United States and Japan in World War II. In 1943, a Japanese admiral was ordered to move a convoy of ships to New Guinea; he had to choose between a rainy northern route and a sunnier southern route, both of which required 3 days sailing time. The Americans knew that the convoy would sail and wanted to send bombers after it, but they did not know which route it would take. The Americans had to send reconnaissance planes to scout for the convoy, but they had only enough reconnaissance planes to explore one route at a time. Both the Japanese and the Americans had to make their decisions with no knowledge of the plans being made by the other side.

   If the convoy was on the route explored by the Americans first, they could send bombers right away; if not, they lost a day of bombing. Poor weather on the northern route would also hamper bombing. If the Americans explored the northern route and found the Japanese right away, they could expect only 2 (of 3) good bombing days; if they explored the northern route and found that the Japanese had gone south, they could also expect 2 days of bombing. If the Americans chose to explore the southern route first, they could expect 3 full days of bombing if they found the Japanese right away but only 1 day of bombing if they found that the Japanese had gone north.

   (a) Illustrate this game in a game table.

   (b) Identify any dominant strategies in the game and solve for the Nash equilibrium.

7. An old lady is looking for help crossing the street. Only one person is needed to help her; more are okay but no better than one. You and I are the two people in the vicinity who can help; we have to choose simultaneously

whether to do so. Each of us will get pleasure worth a 3 from her success (no matter who helps her). But each one who goes to help will bear a cost of 1, this being the value of our time taken up in helping. Set this up as a game. Write the payoff table, and find all pure-strategy Nash equilibria.

8. Two players, Jack and Jill, are put in separate rooms. Then each is told the rules of the game. Each is to pick one of six letters; G, K, L, Q, R, or W. If the two happen to choose the same letter, both get prizes as follows:

Letter	G	K	L	Q	R	W
Jack's prize	3	2	6	3	4	5
Jill's prize	6	5	4	3	2	1

If they choose different letters, each gets 0. This whole schedule is revealed to both players, and both are told that both know the schedules, and so on.

(a) Draw the table for this game. What are the Nash equilibria in pure strategies?

(b) Can one of the equilibria be a focal point? Which one? Why?

9. Suppose two players, A and B, select from three different numbers, 1, 2, and 3. Both players get dollar prizes if their choices match, as indicated in the following table.

		B		
		1	2	3
A	1	10, 10	0, 0	0, 0
	2	0, 0	15, 15	0, 0
	3	0, 0	0, 0	15, 15

(a) What are the Nash equilibria of this game? Which, if any, is likely to emerge as the (focal) outcome? Explain.

(b) Consider a slightly changed game in which the choices are again just numbers but the two cells with (15, 15) in the table become (25, 25). What is the expected (average) payoff to each player if each flips a coin to decide whether to play 2 or 3? Is this better than focusing on both choosing 1 as a focal equilibrium? How should you account for the risk that A might do one thing while B does the other?

10. In Chapter 3, the three gardeners, Emily, Nina, and Talia, play a sequential version of the street-garden game in which there are four distinguishable outcomes (rather than the six different outcomes specified in the example of Section 4.6). For each player, the four outcomes are:
    (i) player does not contribute, both of the others do (pleasant garden, saves cost of own contribution)
    (ii) player contributes, and one or both of the others do (pleasant garden, incurs cost of contribution)
    (iii) player does not contribute, only one or neither of the others does (sparse garden, saves cost of own contribution)
    (iv) player contributes, but neither of the others does (sparse garden, incurs cost of own contribution)

    Of them, outcome i is the best (payoff 4) and outcome iv is the worst (payoff 1). If each player regards a pleasant garden more highly than her own contribution, then outcome ii gets payoff 3 and outcome iii gets payoff 2.
    (a) Suppose that the gardeners play this game simultaneously, deciding whether to contribute to the street garden without knowing what choices the others will make. Draw the three-player game table for this version of the game.
    (b) Find all of the Nash equilibria in this game.
    (c) How might this simultaneous version of the street-garden game be played out in reality?

11. Consider a game in which there is a prize worth \$30. There are three contestants, A, B, and C. Each can buy a ticket worth \$15 or \$30 or not buy a ticket at all. They make these choices simultaneously and independently. Then, knowing the ticket-purchase decisions, the game organizer awards the prize. If no one has bought a ticket, the prize is not awarded. Otherwise, the prize is awarded to the buyer of the highest-cost ticket if there is only one such player or is split equally between two or three if there are ties among the highest-cost ticket buyers. Show this game in strategic form. Find all pure-strategy Nash equilibria.

12. In the film *A Beautiful Mind*, John Nash and three of his graduate school colleagues find themselves faced with a dilemma while at a bar. There are four brunettes and a single blonde available for them to approach. Each young man wants to approach and win the attention of one of the young women. The payoff to each of winning the blonde is 10; the payoff of winning a brunette is 5; the payoff from ending up with no girl is 0. The catch is that, if two or more young men go for the blonde, she rejects all of them and then the brunettes also reject the men because they don't want to be second

choice. Thus, each player gets a payoff of 10 only if he is the sole suitor for the blonde.

(a) First consider a simpler situation where there are only two young men, instead of four. (There are two brunettes and one blonde, but these women merely respond in the manner just described and are not active players in the game.) Show the playoff table for the game, and find all of the pure-strategy Nash equilibria of the game.

(b) Now show the (three-dimensional) table for the case in which there are three young men (and three brunettes and one blonde who are not active players). Again, find all of the Nash equilibria of the game.

(c) Use your results to parts **a** and **b** to generalize your analysis to the case in which there are four young men and then to the case in which there is some arbitrary number, n, of young men. Do not attempt to write down an n-dimensional payoff table. Merely find the payoff to one player when k of the others choose Blonde and $(n-k-1)$ choose Brunette, for $k = 0, 1, \ldots (n-1)$. Can the outcome specified in the movie as the Nash equilibrium of the game—that all of the young men choose to go for brunettes—ever be a true Nash equilibrium of the game?

Appendix: Some General Definitions

We introduced the concepts of dominance and Nash equilibrium in this chapter by using numerical examples. Although this approach may suffice for many of you, some will benefit from knowing more precise definitions. For this purpose,

		COLUMN		
		C1	C2	C3
ROW	R1	R11, C11	R12, C12	R13, C13
	R2	R21, C21	R22, C22	R23, C23
	R3	R31, C31	R32, C32	R33, C33
	R4	R41, C41	R42, C42	R43, C43

FIGURE 4A.1 A General Game