Ch 5

SUMMARY

When players in a simultaneous-move game have a continuous range of actions to choose, best-response analysis yields mathematical *best-response rules* that can be solved simultaneously to obtain Nash equilibrium strategy choices. The best-response rules can be shown on a diagram in which the intersection of the two curves represents the Nash equilibrium. Firms choosing price or quantity from a large range of possible values and political parties choosing campaign advertising expenditure levels are examples of games with *continuous strategies.*

The results of laboratory tests of the Nash equilibrium concept show that a common cultural background is essential for coordinating in games with multiple equilibria. Repeated play of some games shows that players can learn from experience and begin to choose strategies that approach Nash equilibrium choices. Further, predicted equilibria are accurate only when the experimenters' assumptions match the true preferences of players. Real-world applications of game theory have helped economists and political scientists, in particular, to understand important consumer, firm, voter, legislature, and government behaviors.

Theoretical criticisms of the Nash equilibrium concept have argued that the concept does not adequately account for risk, that it is of limited use because many games have multiple equilibria, and that it cannot be justified on the basis of rationality alone. In many cases, a better description of the game and its payoff structure or a refinement of the Nash equilibrium concept can lead to better predictions or fewer potential equilibria. The concept of *rationalizability* relies on the elimination of strategies that are *never a best response* to obtain a set of *rationalizable* outcomes. When a game has a Nash equilibrium, that outcome will be rationalizable; but rationalizability also allows one to predict equilibrium outcomes in games that have no Nash equilibria.

KEY TERMS

best-response curves (127)
best-response rule (124)
continuous strategy (124)
never a best response (145)
rationalizability (145)
rationalizable (145)
refinement (143)

EXERCISES

1. In the political campaign advertising game in Section 1.B, party L chooses an advertising budget, x (millions of dollars), and party R similarly chooses a budget, y (millions of dollars). We showed there that the best-response rules

in that game are $y = \sqrt{x} - x$, for party R, and $x = \sqrt{y} - y$ for party L. Use these best-response rules to verify that the Nash equilibrium advertising budgets are $x = y = 1/4$, or \$250,000.

2. The bistro game of Figure 5.1 defines demand functions for Xavier's (Q_x) and Yvonne's (Q_y) as $Q_x = 44 - 2\,P_x + P_y$, and $Q_y = 44 - 2\,P_y + P_x$. Profits for each firm depend, in addition, on their costs of serving each customer. Suppose, here, that Yvonne's is able to reduce its costs to \$6 per customer by redistributing the serving tasks and laying off several servers; Xavier's continues to incur a cost of \$8 per customer.
   (a) Recalculate the best-response rules and the Nash equilibrium prices for the two firms, given the change in the cost conditions.
   (b) Graph the two best-response curves and describe the differences between your graph and Figure 5.1. In particular, which curve has moved and by how much? Can you account for the changes in the diagram?

3. Yuppietown has two food stores, La Boulangerie, which sells bread, and La Fromagerie, which sells cheese. It costs \$1 to make a loaf of bread and \$2 to make a pound of cheese. If La Boulangerie's price is P_1 dollars per loaf of bread and La Fromagerie's price is P_2 dollars per pound of cheese, their respective weekly sales, Q_1 thousand loaves of bread and Q_2 thousand pounds of cheese, are given by the following equations:

$$Q_1 = 10 - P_1 - 0.5P_2, \quad Q_2 = 12 - 0.5P_1 - P_2.$$

   (a) Find the two stores' best-response rules, illustrate the best-response curves, and find the Nash equilibrium prices in this game.
   (b) Suppose that the two stores collude and set prices jointly to maximize the sum of their profits. Find the joint profit-maximizing prices for the stores.
   (c) Provide a short intuitive explanation for the differences between the Nash equilibrium prices and those that maximize joint profit. Why is joint profit maximization not a Nash equilibrium?
   (d) In this problem, bread and cheese are mutual *complements*. They are often consumed together; that is why a drop in the price of one increases the sales of the other. The products in our bistro example in Section 1.A are *substitutes* for each other. How does this difference explain the differences between your findings for the best-response rules, the Nash equilibrium prices, and the joint profit-maximizing prices in this question, and the corresponding entities in the bistro example in the text?

5. Two carts selling coconut milk (from the coconut) are located at 0 and 1, 1 mile apart on the beach in Rio de Janeiro. (They are the only two coconut

milk carts on the beach.) The carts—Cart 0 and Cart 1—charge prices p_0 and p_1, respectively, for each coconut. Their customers are the beach goers uniformly distributed along the beach between 0 and 1. Each beach goer will purchase one coconut milk in the course of her day at the beach and, in addition to the price, each will incur a transport cost of 0.5 times d^2, where d is the distance (in miles) from her beach blanket to the coconut cart. In this system, Cart 0 sells to all of the beach goers located between 0 and x, and Cart 1 sells to all of the beach goers located between x and 1, where x is the location of the beach goer who pays the same total price if she goes to 0 or 1. Location x is then defined by the expression

$$p_0 + 0.5x^2 = p_1 + 0.5(1 - x)^2.$$

The two carts will set their prices to maximize their bottom-line profit figures, B; profits are determined by revenue (the cart's price times its number of customers) and cost (the carts each incur a cost of \$0.25 per coconut times the number of coconuts sold).

(a) Determine the expression for the number of customers served at each cart. (Recall that Cart 0 gets the customers between 0 and x, or just x, while Cart 1 gets the customers between x and 1, or $1 - x$.)

(b) Write out profit functions for the two carts and find the two best-response rules for their prices.

(c) Graph the best-response rules, and then calculate (and show on your graph) the Nash equilibrium price level for coconuts on the beach.

6. The game illustrated in Figure 5.3 has a unique Nash equilibrium in pure strategies. However, all nine outcomes in that game are rationalizable. Confirm this assertion, explaining your reasoning for each outcome.

7. The game illustrated in Figure 5.4 has a unique Nash equilibrium in pure strategies. Find that Nash equilibrium, and then show that it is also the unique rationalizable outcome in that game.

8. Section 4.B describes a fishing game played in a small coastal town. When the response rules for the two boats have been derived, rationalizability can be used to justify the Nash equilibrium in the game. In the description in the text, we take the process of narrowing down strategies that can never be best responses through three rounds. By the third round, we know that X (the number of barrels of fish brought home by boat 1) must be at least 9, and that Y (the number of barrels of fish brought home by boat 2) must be at least 4.5. The narrowing process in that round restricted X to the range between 9 and 13.75 while restricting Y to the range between 4.5 and 7.5. Take this process of narrowing through one additional (fourth) round and show the reduced ranges of X and Y that are obtained at the end of the round.

9. Nash equilibrium through rationalizability can be achieved in games with upward-sloping best-response curves if the rounds of eliminating never-best-response strategies begin with the smallest possible values. Consider the pricing game between Xavier's Tapas Bar and Yvonne's Bistro that is illustrated in Figure 5.1. Use Figure 5.1 and the best-response rules from which it is derived to begin rationalizing the Nash equilibrium in that game. Start with the lowest possible prices for the two firms and describe (at least) two rounds of narrowing the set of rationalizable prices toward the Nash equilibrium.

10. **Optional, requires calculus** Recall the political campaign advertising example from Section 1.B concerning parties L and R. In that example, when L spends \$$x$ million on advertising and R spends \$$y$ million, L gets a share $x/(x+y)$ of the votes and R gets $y/(x+y)$. We also mentioned that two types of asymmetries can arise between the parties in that model. One party—say, R—may be able to advertise at a lower cost or R's advertising dollars may be more effective in generating votes than L's. To allow for both possibilities, we can write the payoff functions of the two parties as

$$V_L = \frac{x}{x+ky} - x \quad \text{and} \quad V_R = \frac{ky}{x+ky} - cy.$$

These payoff functions show that R has an advantage in the relative effectiveness of its ads when k is high and that R has an advantage in the cost of its ads when c is low.

(a) Use the payoff functions to derive the best-response functions for R (which chooses y) and L (which chooses x).

(b) Use your calculator or your computer to graph these best-response functions when $k = 1$ and $c = 1$. Compare the graph with the one for the case in which $k = 1$ and $c = 0.8$. What is the effect of having an advantage in the cost of advertising?

(c) Compare the graph from part b, when $k = 1$ and $c = 1$ with the one for the case in which $k = 2$ and $c = 1$. What is the effect of having an advantage in the effectiveness of advertising dollars?

(d) Solve the best-response functions that you found in part a, jointly for x and y, to show that the campaign advertising expenditures in Nash equilibrium are

$$x = \frac{ck}{(c+k)^2} \quad \text{and} \quad y = \frac{k}{(c+k)^2}.$$

(e) Let $k = 1$ in the equilibrium spending level equations and show how the two equilibrium spending levels vary with changes in c. The let $c = 1$ and show how the two equilibrium spending levels vary with changes in k. Do your answers support the effects that you observed in parts b and c of this exercise?