Maths Revision

From the document, "MBA Maths Unit 3 Functions & Interests (sic)" pages 46/47: Functions — Quadratics.

A quadratic function constains a variable squared:

$$y = ax^2 + bx + c$$

The (single) maximum or minimum of y always occurs at:

$$x = -\frac{b}{2a}$$

We can see this by differentiating y: $\frac{dy}{dx} = 2ax + b$, which is zero to maximize y, at $x^* = -b/2a$.

Whether y is a maximum or minimum depends on the sign of the coefficient a:

when a is positive, y is a minimum,
when a is negative, y is a maximum,
and when a is zero, y is a linear function in x.

>

Let's find the profit-maximizing values in L5.

p.5 *Von Stackelberg: Reaction Function*

$\pi_C = (10 - (Q_S + Q_C)) \times Q_C - 3Q_C$

$\therefore\ \pi_C = -Q_C^2 + (7 - Q_S) \times Q_C + 10$ is a quadratic in Q_C,
with $a = -1$, $b = (7 - Q_S)$, and $c = 10$.
$\therefore\ \pi_C$ max at $Q_C^* = -\frac{b}{2a} = \frac{7 - Q_S}{2}$

p.6 *Von Stackelberg: Solution*

$\pi_S = -\frac{1}{2} Q_S^2 + 3\,\frac{1}{2}\,Q_S$ is a quadratic in Q_S,
with $a = -½$, $b = 3½$, and $c = 0$.
$\therefore\ \pi_S$ max at $Q_S^* = -\frac{b}{2a} = 3\,\frac{1}{2}$.

p.8 *Monopolist: Solution*

$\pi_M = P_M Q_M - 3Q_M = Q_M^2 + 7Q_M$ is a quadratic in Q_M,
with $a = -1$, $b = 7$, and $c = 0$.
$\therefore\ \pi_M$ is max at $Q_M^* = 7/2$.

p.11 ***Cournot: Reaction Function***

$\pi_S = (10 - Q_C - Q_S) \times Q_S - 3Q_S$

$\pi_S = -Q_S^2 + (7 - Q_C)Q_S$ is a quadratic in Q_S,

with $a = -1$, $b = 7 - Q_C$, and $c = 0$.

$\therefore\ \pi_S$ is a max at $Q_S^* = \frac{7 - Q_C}{2}$.

p.15 ***Imperfect Bertrand: Solution***

$\pi_p = -P_p^2 + (30 + \frac{1}{2} P_d) \times P_p - 144 - 3P_d$ is a quadratic in P_p,

with $a = -1$, $b = 30 + \frac{1}{2} P_d$, and $c = -144 - 3P_d$

$\therefore\ \pi_p$ is max at $P_p^* = 15 + \frac{1}{4} P_d$

p.26 ***Monopolistic Cartel: Solution***

$\pi_M = (10 - Q_M) \times Q_M - 1 \times Q_M = -Q_M^2 + 9Q_M$ is a quadratic in Q_M,

with $a = -1$, $b = 9$, and $c = 0$.

$\therefore\ \pi_M$ is max at $Q_M^* = 9/2 = 4\ ½$.

p.27 ***Cournot: Reaction Function***

$\pi_2 = (10 - y_2 - y_1^e) \times y_2 - 1 \times y_2$

$\pi_2 = -y_2^2 + (9 - y_1^e) \times y_2$ **is a quadratic in** y_2**,**

with $a = -1$**,** $b = 9 - y_1^e$**, and** $c = 0$**.**

$\therefore$ π_2 **is max at** $y_2^* = \frac{1}{2}(9 - y_1^e)$**.**

p.28 ***Von Stackelberg: Solution***

$\pi_1 = (10 - y_2 - y_1) \times y_1 - 1 \times y_1$

$\pi_1 = (10 - \frac{1}{2}(9 - y_1) - y_1) \times y_1 - y_1$

$\pi_1 = 10y_1 - \frac{9}{2} y_1 + \frac{1}{2} y_1^2 - y_1^2 - y_1$

$\therefore$ $\pi_1 = 4\frac{1}{2} y_1 - \frac{1}{2} y_1^2$ **is a quadratic in** y_1**,**

with $a = -½$ **,** $b = 4½$**, and** $c = 0$**.**

$\therefore$ π_1 **is max at** $y_1^* = 4½$**.**

The Economics of Profit-Maximizing

The profit-maximizing firm will continue increasing its rate of output until the revenue associated with selling another unit of output is equal to or less than the cost of producing that output, assuming rising costs.

These two measures are called the "Marginal Revenue" and the "Marginal Cost", respectively.

The difference between Marginal Revenue and Marginal Cost is the Marginal Profit associated with the last unit of output produced and sold.

In algebra: $M\pi = MR - MC$, where all three are functions of the level of output Q (amongst other things, such as the demand curve, or the going market price P).

With rising costs, profit is maximized when Marginal Profit is no longer positive: when Total Profit no longer rises with the rate of production.

Or, when $M\pi = MR - MC = 0$.

If we have functions for MR and MC in terms of output Q, then we can determine the profit-maximizing level of output Q^*.

Marginal Revenue and Cost are just the derivatives of Total Revenue and Total Cost with respect to output Q.

TR is just $P \times Q$. With a linear demand curve (say $P = 10 - Q$), and a firm with some *market power* (which means can set its own price, subject to demand), MR can be calculated:

$TR = P \times Q = (10 - Q) \times Q = 10Q - Q^2$.

Differentiating with respect to output Q:

$$\frac{dTR}{dQ} = MR[Q] = 10 - 2Q$$

Let's say that it costs a constant amount (say, \$3) to produce and sell a unit of output. Then $TC = 3Q$, and $MC = 3$.

For this firm, choosing Q^* to equate MR and MC will result in the higest profit:
solve: $10 - 2Q = 3$ to get $Q^* = 3½$.

From the demand function, with $Q^* = 3½$, the price P will be \$6.50: the higher the output, the lower the price to sell all units.

Profit-Maximizing, Graphically

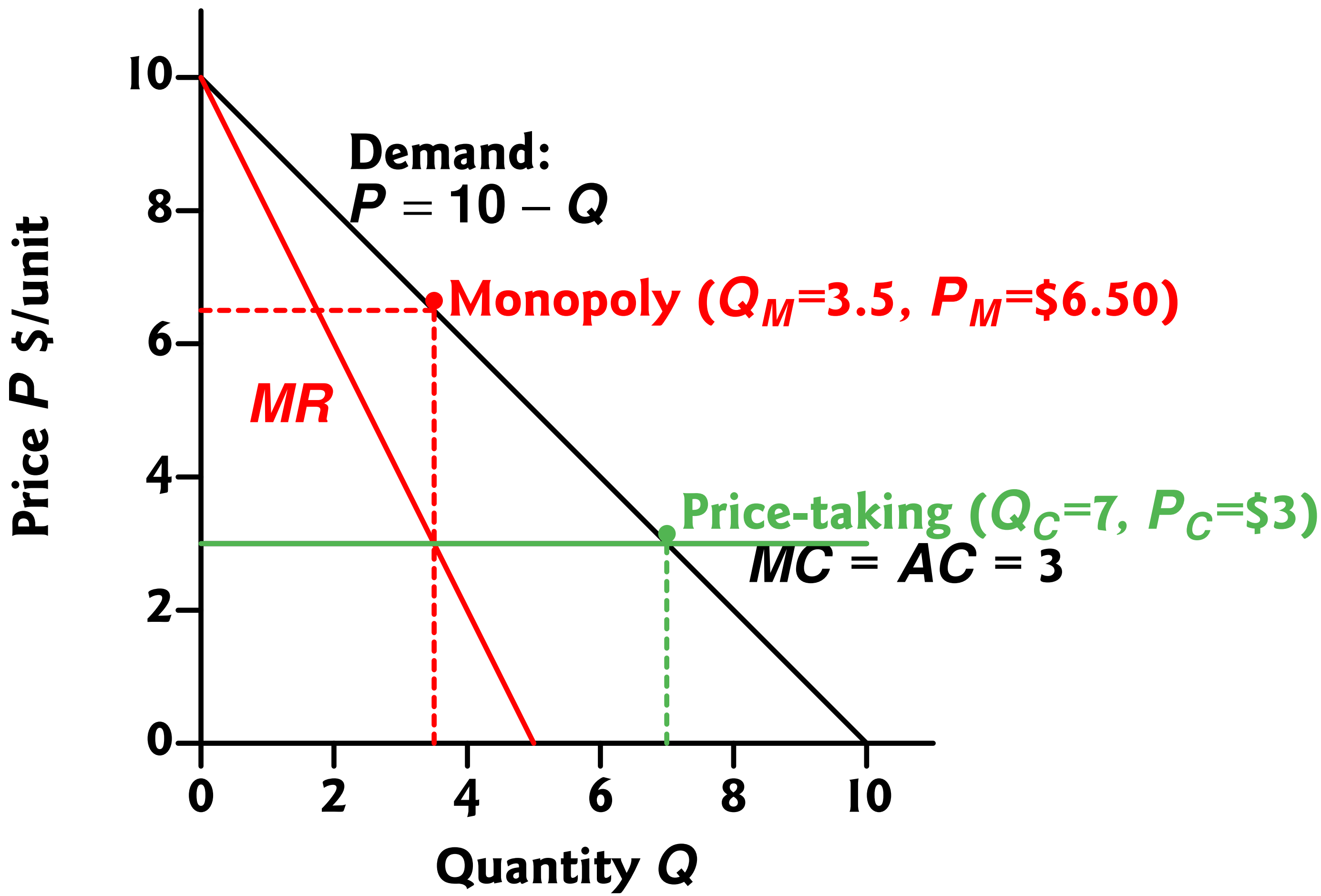
We can plot the MC = \$3/unit line and the demand line $P = 10 - Q$. We can identify the Monopolist's price & quantity (P_M, Q_M) and the price-taker's price & quantity $(P_{C,} Q_C)$.

Monopolists's
$TR = P_M \times Q_M = (10 - Q_M) \times Q_M = 10Q_M - Q_M^2$

Differentiating TR, we get
$MR = \frac{dTR}{dQ_M} = 10 - 2Q_M$, which is exactly twice as steep as the demand line, as shown below.

From above, profit-maximizing monopolistic output Q_M^* occurs when MR = MC, but here MC = \$3.
So Q_M^* from $10 - 2Q_M^* = 3$, or $Q_M^* = 3\,\frac{1}{2}$.

The diagram shows that the output Q_M^* = 3½ occurs where the red downwards-sloping MR line cuts the MC = \$3 line, and reading up to the demand line gives P_M^* = \$6.50.


Demand:
P = 10 – Q
Monopoly (Q_M=3.5, P_M=$6.50)
MR
Price-taking (Q_C=7, P_C=$3)
MC = AC = 3
Price P $/unit
Quantity Q
0
2
4
6
8
10

If the firm is behaving as a price-taker, then its Marginal Revenue is just the going price P.

So it chooses its output where its Marginal Revenue = \$3, the Marginal Cost.

The Marginal Cost = \$3/unit is in effect the market Supply curve.

The market quantity is $Q_C = 7$, at P_C = \$3/unit, where Demand = the horizontal green Supply line, as shown above.