Combining Simultaneous and Sequential Games

Let's mix and match our games, explore how games can change, and how we can model them:

1. **Simultaneous and Sequential Together.**
2. **Changing the Order of Moves:**
   - **First-Mover Advantage**
   - **Second-Mover Advantage**
   - ***Both*-Mover Advantage**
3. **Trees for Simultaneous Games.**
4. **Matrices for Sequential Games.**
   - **Subgame Perfect Equilibrium**

1. Games with Both

Players: CrossTalk (CT) and GlobalDialog (GD)

Actions: each Invest $10 b in a separate fibre-optic network or not, simultaneously.

Neither invests: end of game.

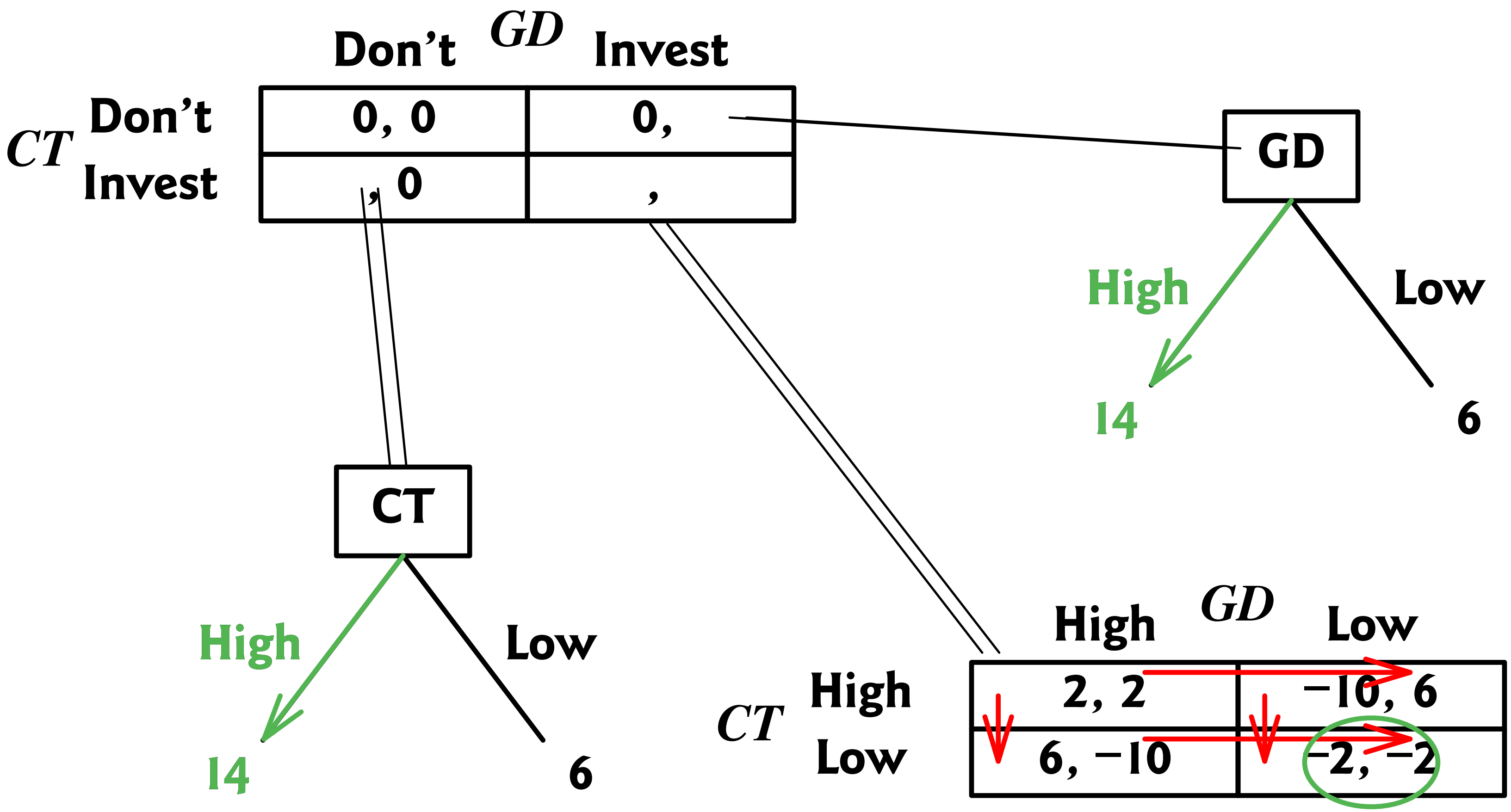
Only one invests: it must choose its price:

- **price High (60 m customers, $400/cust rev), or**
- **price Low (80 m cust, $200/cust).**

Both invest: second simultaneous game:

- **both price High (each 30 m cust, $400/cust),**
- **both price Low (each 40 m cust, $200/cust), or**
- **one High and the other Low (High gets 0 cust, Low gets 80 m cust, $200/cust).**

A Two-Stage Game ($k)



Rolling Back the Game

CT's payoff if it alone Invests and prices High = \$14 b

$= \$400 \times 60\text{ m} - \$10\text{ b}$
$= \$24\text{ b} - \$10\text{ b}$
$= \$14\text{ b}$

If both Invest and price Low, each gets −\$2 b

$= \$200 \times 40\text{ m} - \$10\text{ b}$
$= \$8\text{ b} - \$10\text{ b}$
$= -\$2\text{ b}$

Etc.

The second-stage pricing game is a PD: pricing Low is dominant.

So what is the entire game?

Hence, after rollback, the entire game is:

		GD Don't	*GD* Invest
CT	Don't	0, 0	0, 14
CT	Invest	14, 0	−2, −2

A Chicken! game.

What if one could move first? — see later.

A Subgame

The second-stage simultaneous pricing game at the bottom right of Page 3 is a complete game on its own.

It is also a *subgame* of the full game.

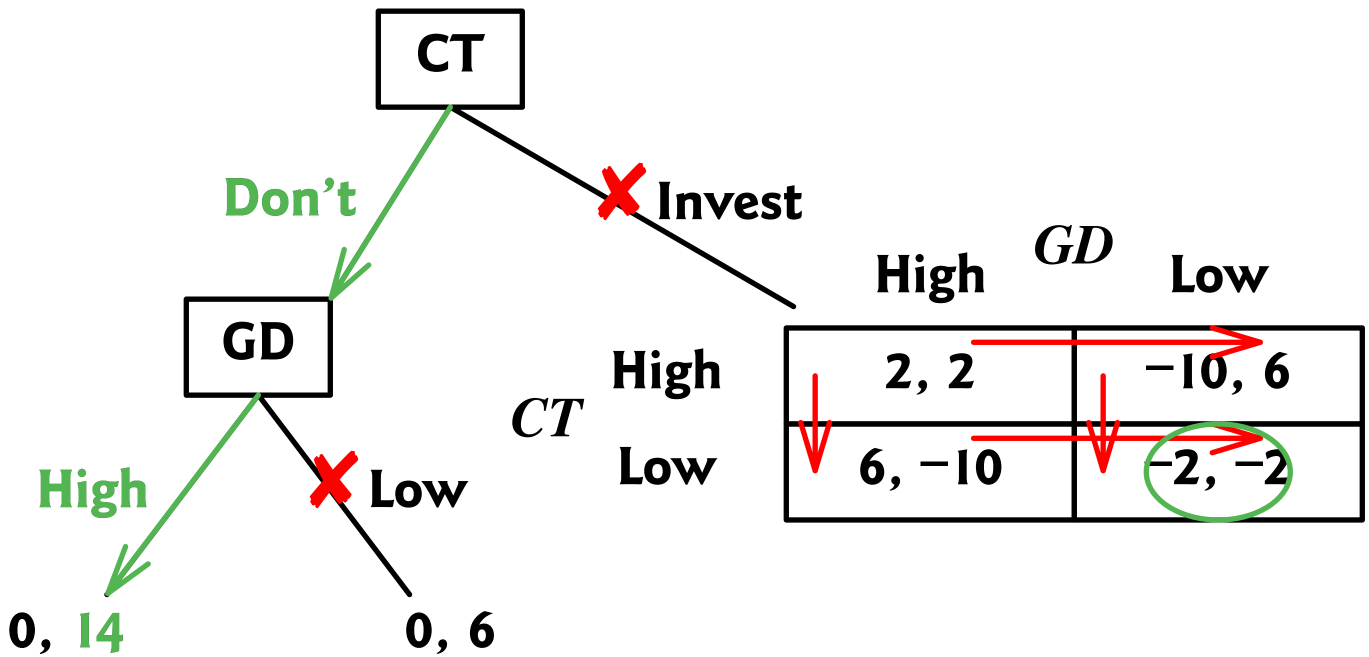
A subgame: part of a multi-move game that begins at a particular decision node of the original (larger) game.

A multi-move game has as many subgames as it has decision nodes □.

Later: subgame perfect equilibrium (SPE) and N.E., and the importance of *credible strategies*.

Sequential then Simultaneous

**What if GD has already Invested $10 b and CT knows it?
Or GD has made a credible commitment to Invest?
CT now has to decide whether to Invest; then the pricing decision is made.**



**∴ CT Doesn't Invest, or may try to get in first.
See Lect. 14: *strategic moves*, and exA: Hold-Up.**

2. Changing the Players' Order of Moves

No Change In Outcome

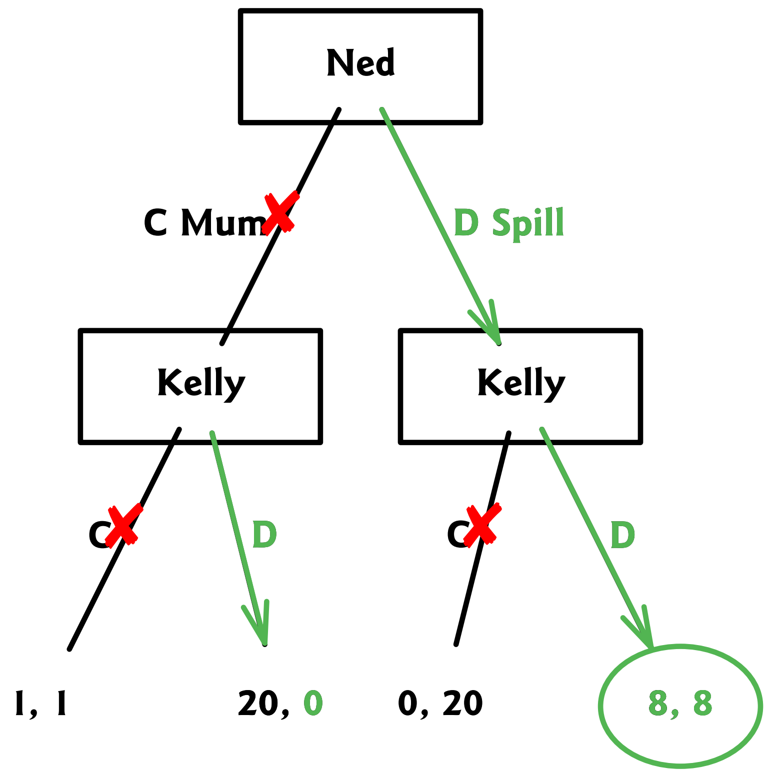
When *both or all players have dominant strategies*, there is no change in outcomes. There is no advantage in moving first, or second. See the PD below.

The PD POM: Years of prison (Ned, Kelly):

		Kelly	
		D Spill	**C Mum**
Ned	**D Spill**	**8, 8**	**0, 20**
	C Mum	**20, 0**	**1, 1**

Fewer years in prison are better: D,D is the N.E. (8,8).

PD: Ned moves first:



Again: the N.E. is D,D or Spill,Spill, as with the simultaneous game.
(No difference if Kelly moves first.)

∴ No first- or second-mover advantage in a PD.

2a. First-Mover Advantage

See the Capacity Game in Lectures 2 & 4, and

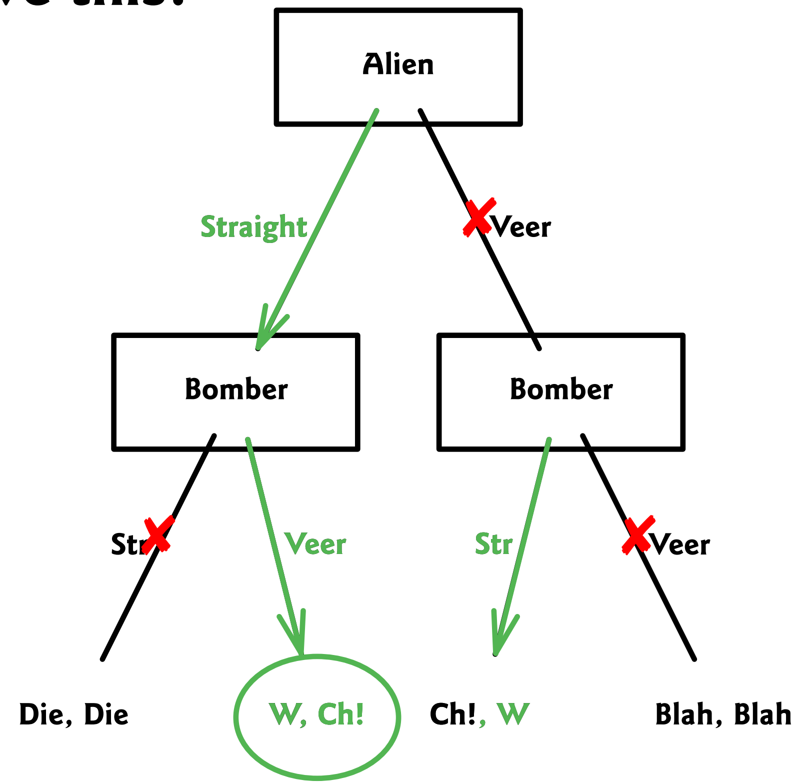
Here: Chicken!

		Bomber	
		Veer	**Straight**
Alien	**Veer**	**Blah, Blah**	**Chicken!, Winner**
	Straight	**Winner, Chicken!**	**Death? Death?**

Two N.E., but no easy way to coordinate on one or the other.

Chicken: Alien Moves First:

How do we solve this?



No longer two N.E.: only one: First Mover — go Straight, Second Mover — Veer.
∴ a clear first-mover advantage.

(So: how to commit to Straight credibly?)

2b. Second-Mover Advantage

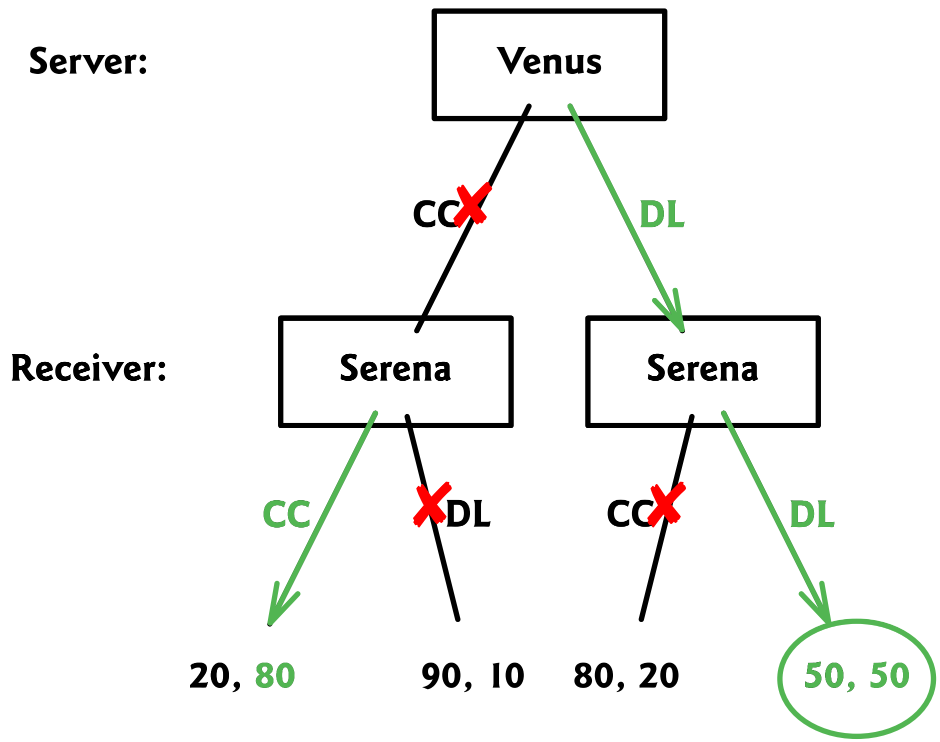
The Tennis game between Venus serving and Serena receiving from Lecture 2 has the POM:

		Serena	
		DL	**CC**
Venus	**DL**	**50**	**80**
	CC	**90**	**20**

Whichever action Venus chooses with her serve, and whichever action receiver Serena chooses in her court coverage, one or the other will regret the combination: ∴ no N.E. (in pure strategies).

Venus the server moves first

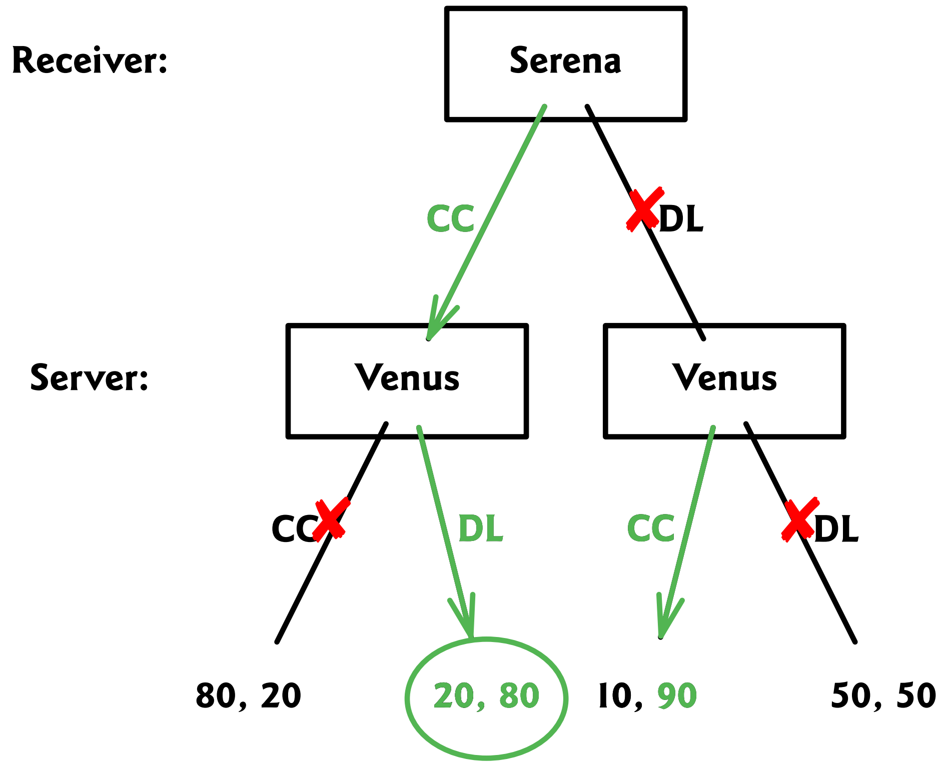
But if receiving Serena can pick serving Venus's choice in time:



Serena has the second-mover advantage, and wins the shot half (50%) the time. Venus wins 50% too.

Serena the receiver moves first

Or if Venus serving can pick Serena's choice in time:



Venus has the second-mover advantage, and wins the shot most (80%) of the time.

2c. *Both* Players May Do Better

The Macro game from Lecture 2 has the POM:

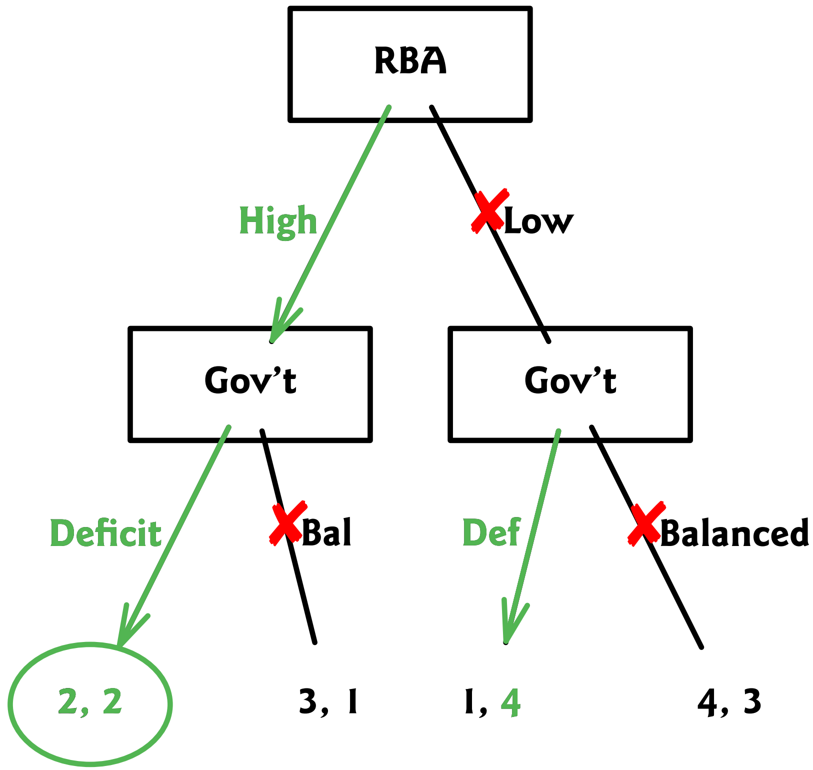
		RBA	
		Low	**High**
Gov't	**Balanced**	3, 4	1, 3
	Deficit	4, 1	2, 2

The Gov't has a dominant strategy of Deficit, which the RBA knows, and ∴ chooses High interest rates.

Yields payoffs of 2 (the second-worst outcome) for each.

But What If The RBA Moves First?

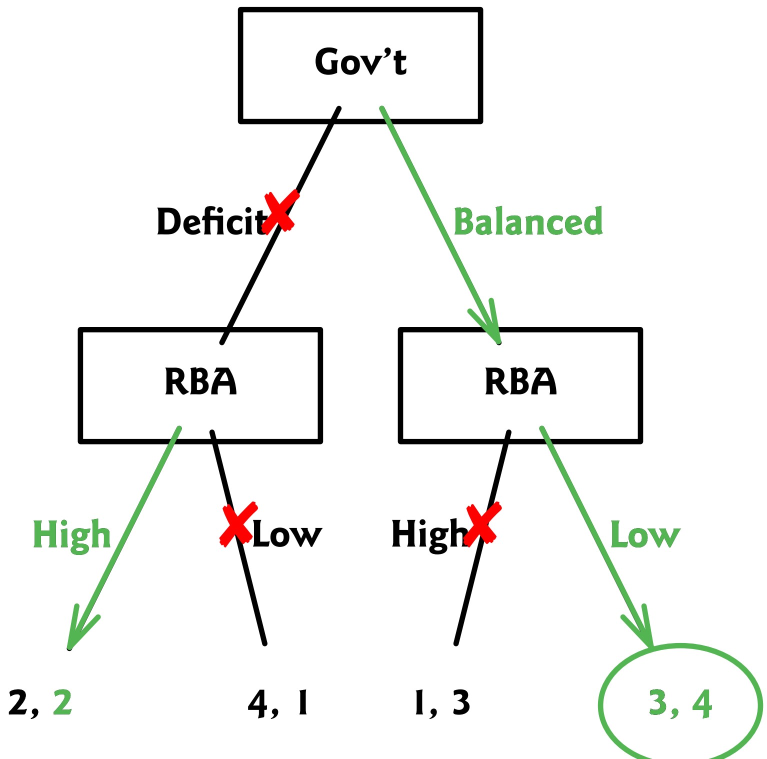
The game tree (4 = best, 1 = worst):



The RBA knows that the Gov't will go into Deficit, come what may, and so chooses High interest rates, yielding the RBA 2 instead of 1. As in the simultaneous game.

But if the Gov't moves first:

The game tree is:



The choosen combination of strategies is {Balanced, Low}: this is the Rollback Equilibrium (R.E.), and, surprisingly, yields a better outcome for *both* players than does {Deficit, High}.

How?

But Balanced is a *dominated* strategy for the Gov't: how is it part of the R.E. in the sequential-move game in which the Gov't moves first?

The Gov't knows that:

if it chooses Deficit, the RBA will choose High; and

if it chooses Balanced, the RBA will choose Low.

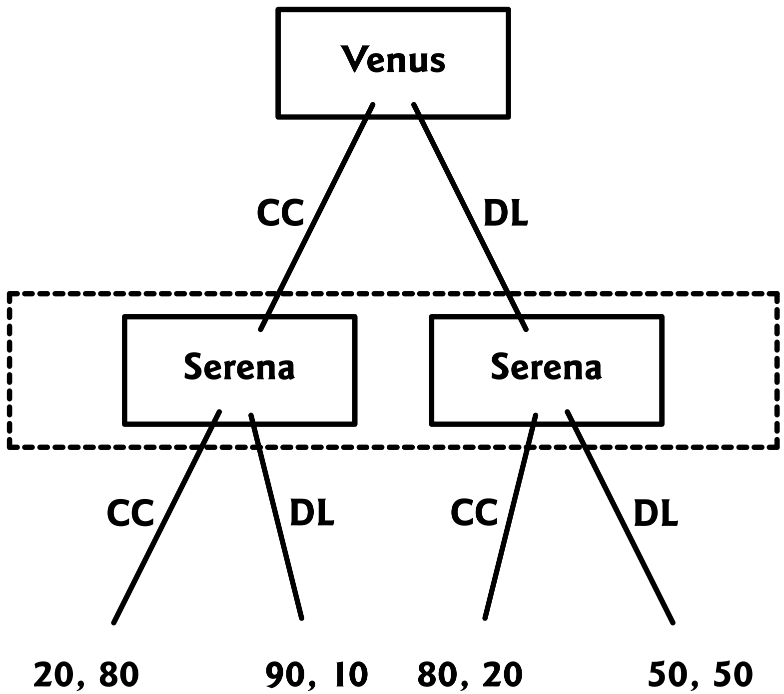
The Gov't prefers {Balanced, Low} to {Deficit, High} — all those mortgages! — so Gov't gets its second-best outcome, and the RBA its best outcome.

The Gov't must know that the RBA has the *flexibility* to respond (with Low).

(See also the Capacity Game Revisited, Lecture 4, pp. 6–8, where a dominated move — Large — is also chosen in R.E.)

3. Trees for Simultaneous Games

The simultaneous Tennis game as a tree:



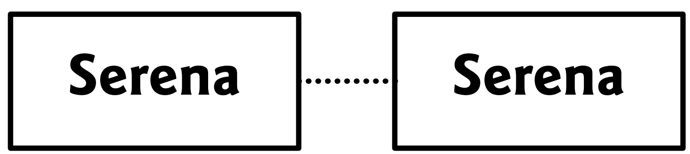
The dotted box is an *Information Set:* Serena can't tell which of the two decision nodes she's at since she doesn't (yet) know how Venus will serve (CC or DL) and so she cannot do CC at one and DL at the other — *there can only be 1 action per Info Set.* (DSkR p.194)

Information Sets

Serena must choose without knowing what Venus has picked: Serena doesn't know which decision node she's at.

Use a dotted box around the relevant decision nodes to indicate her *lack of specific information.*

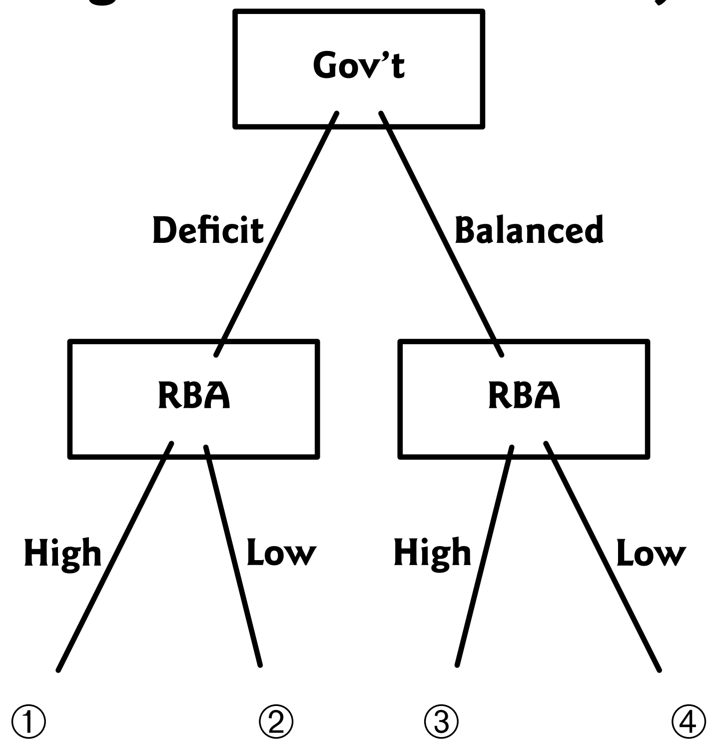
Information Sets could also be called "ignorance sets," since the player doesn't know what's happened, or where she is exactly in the game tree. An alternative convention is to join the decision nodes with a dotted line:



So a *strategy*: a complete plan of action, specifying the move a player would make at each Information Set (instead of each decision node) when the rules of the game specify that it is her turn to move.

4. Matrices for Sequential Games

Use the Macro game tree, where the Gov't moves first. (Instructions for *delegation* of RBA action.)



Four possible strategies for the RBA:

first, ① & ③: always H; second, ② & ④: always L; third, ④ & ①: L if B & H if D; and fourth, ③ & ②: H if B & L if D.

As a payoff matrix:

		RBA			
		L if B & H if D	H if B & L if D	L always	H always
Gov't	Bal	3, 4	1, 3	3, 4	1, 3
	Def	2, 2	4, 1	4, 1	2, 2

4: best; 1: worst

Gov't's possible strategies: Balanced or Deficit.

RBA has four possible strategies: always High; always Low; Low if Balanced and High if Deficit (L if B & H if D); High if Balanced and Low if Deficit (H if B & L if D).

The last two columns are as if the game were simultaneous, but in the first two columns RBA's decision depends on Gov't's.

Using arrow or cell-by-cell, find two NE:

1. **{Balanced, L if B & H if D} with payoffs (3,4), found by rollback on page 17 above, and**
2. **{Deficit, H always} with payoffs (2,2).**

Why are there two N.E. in this analysis of the sequential game but only one using rollback (R.E.) (p.16)?

N.E. when: neither player gains from moving, given the other's strategy. But R.E. asks: what would the player do, at each decision node (or Info Set)?

"H always" is not optimal at one decision node: if Gov't chose Balanced, then RBA chooses Low (4 preferred to 2). So R.E. can't include "H always".

But the N.E. doesn't do the R.E. test: "H always" and Deficit are mutual best responses ("H always" equal to "L if B & H if D").

SPE, however, excludes non-credible strategies.

Subgame-Perfect Equilibrium SPE

R.E. requires that all players make their best choices in *every* subgame of the larger game, whether or not along the equilibrium path down the tree.

Strategies are complete courses of action: for each and every decision node of the tree, on or off the equilibrium path.

The 2nd N.E. {Gov't Deficit, then RBA High} is on the equilibrium path.

But {Gov't Balanced, then RBA High} (from "H always") is not optimal for this off-equilibrium subgame.

{Gov't Balanced, then RBA Low} (from "L if B & H if D") is optimal along the equilibrium path, and off the path too (Deficit provokes High).

∴ "H always" lacks *credibility*: it is not in the R.E. and ∴ is not in SPE.