7. Gaining Insight

The product of any analysis should be new insights which clarify a course of action. **The process of evaluation has three parts: (See Clemen, Package.)**

7.1 *Deterministic evaluation*

- **Sensitivity analysis**
- **Tornado diagrams**

7.2 *Probabilistic evaluation*

- **Cumulative probability distribution**
- **Sensitivity to probability**

7.3 *Expected Value of Information*

- **Expected Value of Perfect Information (VPI)**
- **Expected Value of imperfect information**

7.1 Deterministic Evaluation

Deterministic evaluation may be closest to the way most analyses are performed outside decision analysis.

Sensitivity analysis provides the ability to determine the most important factors which affect either the decision or the value ("the bottom line"). We can then use the *Tornado diagram* to illustrate the relative sensitivities of each variable.

Variables for Glix:

	Probabilities		
	10	50	90
Market size (Gigagrams)	0.2	1	2
Market share (%)	15	20	25
Mfg. Costs ($/kg)	1	1.5	2
Mktg. Costs ($/kg)	0.5	0.75	1

↑

baseline

Conducting sensitivity analysis on uncertainties:

Step 1: **Build a deterministic value model** which uses the uncertainties identified in the frame and calculates according to the decision criterion.

Step 2: **Choose** a low (10th percentile — a 10% chance of the variable falling below X), base (50%), and high (90%) value for each uncertain event.

Step 3: **Run the model with all uncertainties set at their base values**, and record the calculated value.

Step 4: **Run the model swinging each variable** from its 10th percentile to its 90th, while holding all other variables at their base values. Record the calculated value at each setting.

Step 5: **Plot a Tornado diagram** using the data.

Building the value model

The value model for Glix:

Fixed inputs:

- **Discount rate = 10% p.a.**
- **Tax rate = 40%**
- **Glix price/kg = $5.00**
- **Project length = 10 years**

NPV of Glix = (Revenue − Cost)× Discount Factor for each year

Revenue = Price × Volume

Volume = Market Size × Market Share

Cost = (Manufacturing Cost + Market Cost) × Volume

Base case value for Glix:

Revenue = $\$5.00 \times 1{,}000{,}000\ \text{kg} \times 20\%$

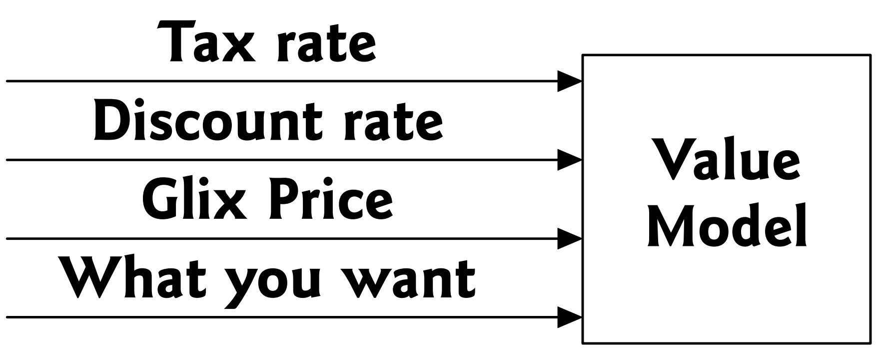
Costs $\rightarrow$ $(\$1.50 + \$0.75) \times 1{,}000{,}000$

$\rightarrow$ **$\$1{,}000{,}000 - \$450{,}000$**

$\rightarrow$ **$\$550{,}000/\text{year} \times (1-0.40)$ after tax**

$\rightarrow$ **$\$330{,}000 \times 10\ \text{years} \times 10\%$**

$\therefore$ **Profit = $\$1{,}209{,}525$**



Plot the Tornado using graph paper or software.

Step 1: Calculate the swing of each variable, from the 10th to the 90th percentile.

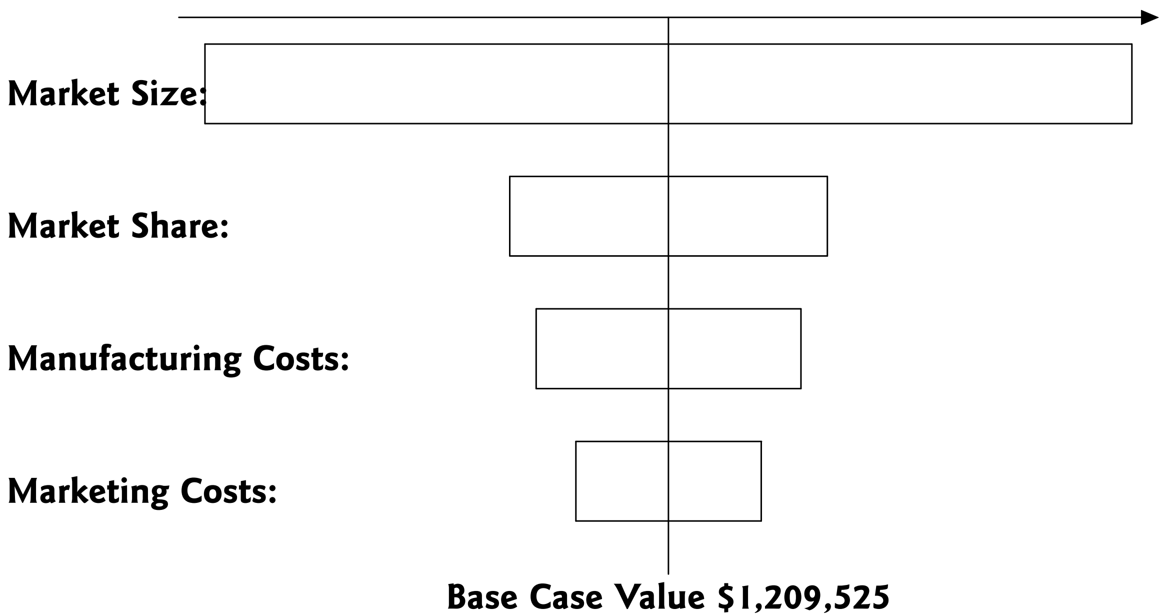
Step 2: Rank in order the swings in value from largest to smallest.

Step 3: Draw a horizontal line and determine an appropriate value scale.

Step 4: Draw a vertical line which cuts the horizontal line at the base case value.

Step 5: Draw horizontal bars for each uncertainty relative to their swings in value.

The Tornado Diagram.



See Clemen (Reading 18) and Skinner (Reading 20) for further discussion.

Simplifying the model:

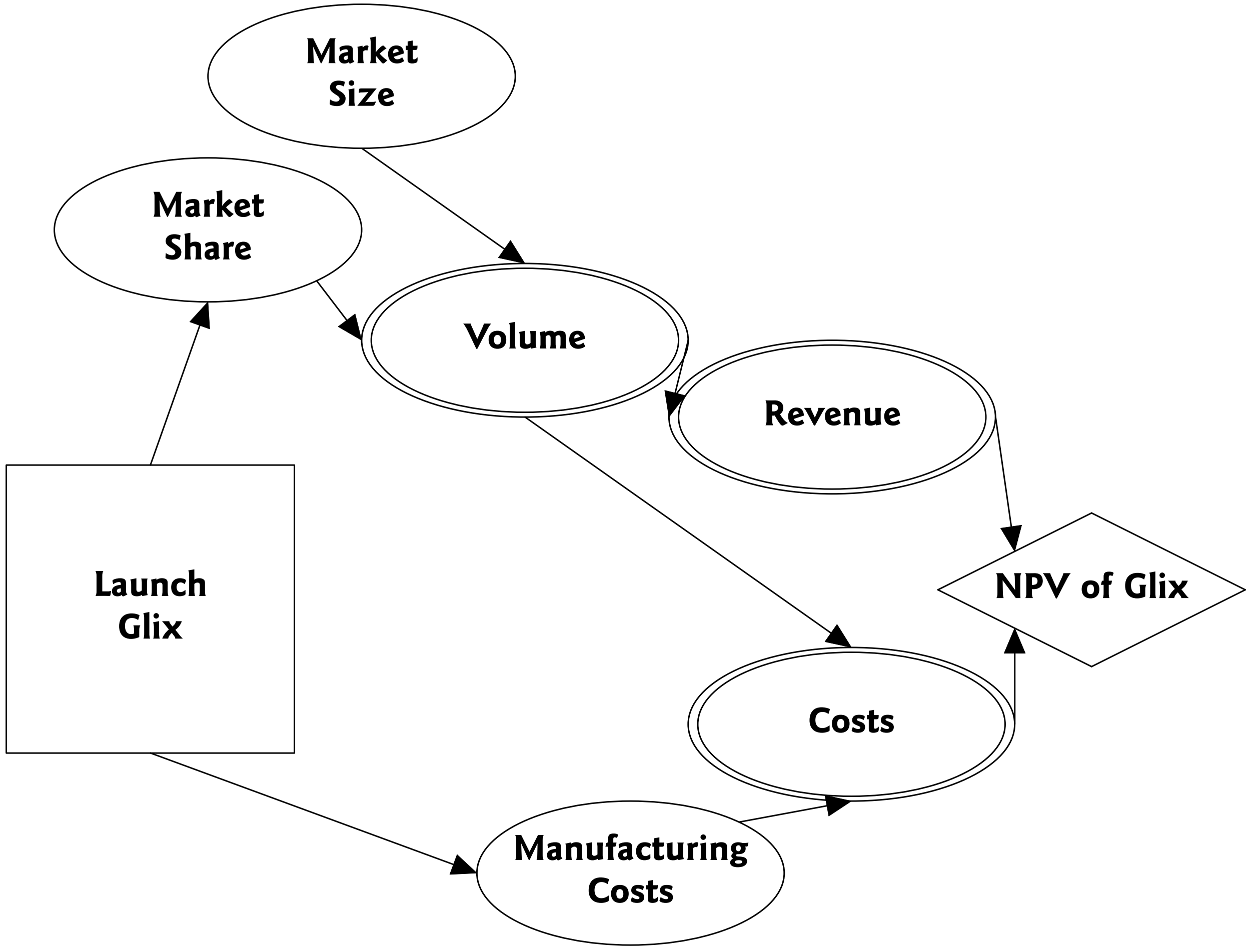
Tornado diagrams provide insight into the key uncertainties affecting the decision.

The decision model can then be *simplified* using the insights gained from the sensitivity analysis. *This is very important for large models with many uncertainties.*

With project Glix, the most important uncertainty is Market Size, and the least important is Marketing Costs, from the Tornedo Diagram above.

***Important:* always strive to simplify your Influence Diagrams: use Tornado diagrams and your intuition to reduce the degree of complexity of the ID — they are much more useful when simple!**

Influence Diagram of Glix Decision



7.2 Probabilistic Evaluation

Deterministic uncertainty is important for identifying key variables but does not provide insight into the likelihood of any scenario.

The cumulative probability distribution provides a graphical risk profile for the project or each alternative.

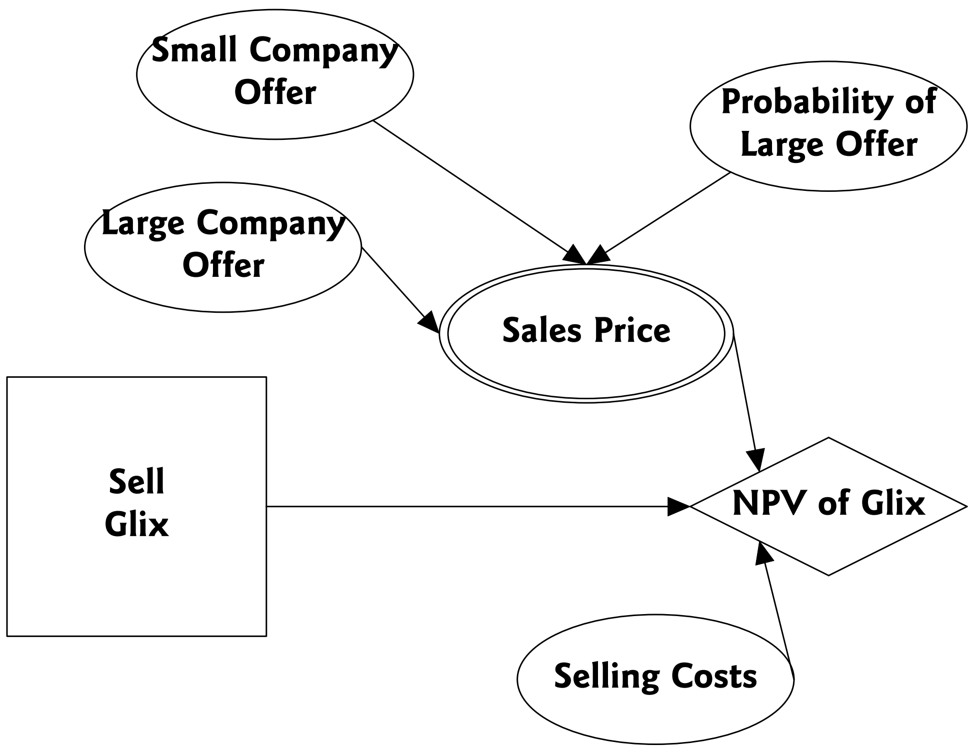
(This is more technical: see David C. Skinner, *Introduction to Decision Analysis* (Gainesville, Fl., 2nd. ed., 1999), pp. 112–113, 218–220.)

But see Laura's decision below.

Another alternative? Selling the Glix project.

In addition to launching Glix, the company also wanted to evaluate the alternatives of selling and/or licensing the product.

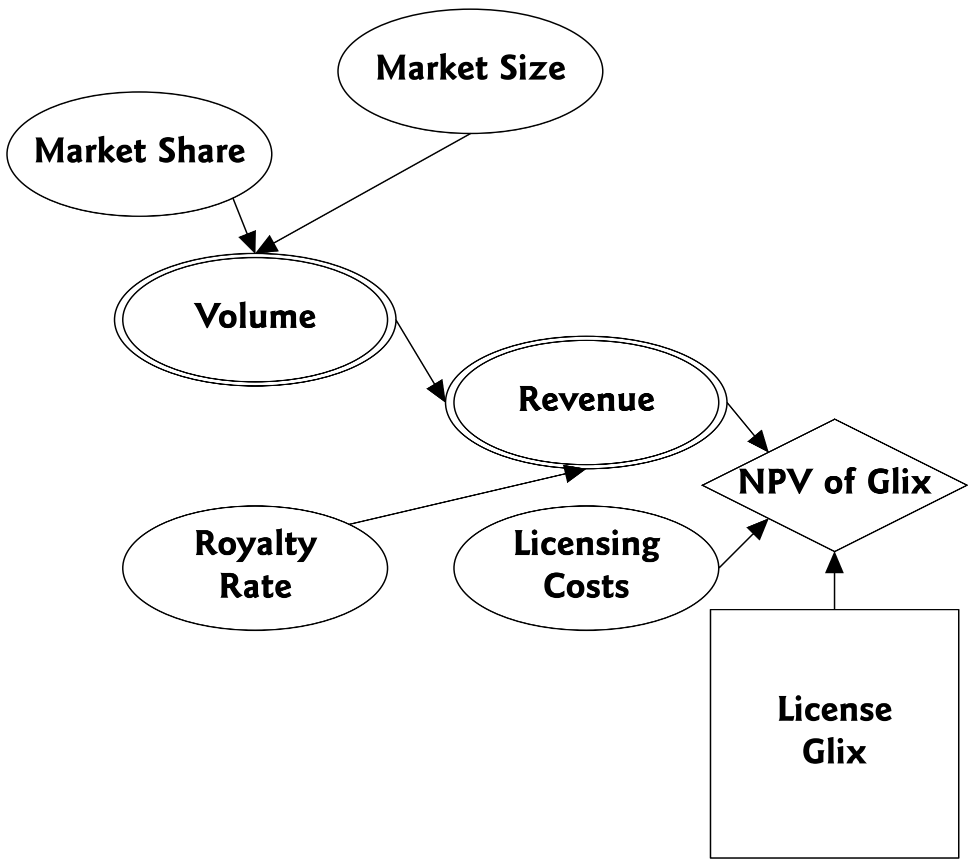
The influence diagram for selling Glix to another company:



The EMV of Selling = \$320,000.

Or Licensing Glix:

The company could license Glix and receive royalties from the sales.



The EMV of Licensing = $1,135,000

Comparing alternatives

We can compare each alternative on a consistent basis, thereby fully examining the risk and opportunity of each alternative.

Choosing wisely:

Dominance—

- **Dominance can be *deterministic* or *stochastic***
- **Allows inferior alternatives to be eliminated**
- **Is always better than the other alternatives**

It turns out, with further analysis, that none of the three alternatives shows complete dominance over the other two.

The "sell" alternative, however, is less attractive, based on an EMV of $320,455.

Sensitivity to probability:

Sensitivity to probability is similar to deterministic sensitivity analysis in that the aim is to identify variables which would change the decision.

Having said that any subjective probability which incorporates the expert's available knowledge, beliefs, experiences, and data is correct, we need to know how sensitive the decision is to any particular probability. This will help us choose between launching or licensing Glix.

It turns out that we should launch if we are confident that *launching* has a greater than 40% chance of success.

Games Against Nature: Gaining Insight — The Value of Information

Today's topics:

1. The Value of Perfect Information
   a. For Laura
   b. For Glix
2. Probabilistic Sensitivity Analysis
   a. For Laura
3. The Value of Imperfect Information
   a. For Laura
   b. For Glix

1. The Value of Information

We can determine the value of gathering additional information before spending time or money to gather it.

The *Value of Perfect Information* is the easiest to calculate, and provides an upper boundary as to the *most* we *should* ever spend on new information.

Most companies over-invest in information, spending more than it is worth to them.

The Value of Perfect Information (VPI) is the *most* that we should spend for new information which is not 100% reliable.

We would only value Perfect Information if it changed our decisions, otherwise not.

1a. Laura's Case — The Expected Value of Perfect Information (VPI)

Laura could reduce uncertainty through *information gathering*:

- **Laura could employ a market-research firm to test for the acceptance and demand for Retro.**
- **If totally reliable (no errors), then**
  - **if "Retro is definitely a Goer", then a return of $240k, less the price of the Trial**
  - **if the Trial indicates Retro is a "Fizzer," then choose a net return of $200k with Trad, less the price of the Trial**

Laura has two decisions to make:

1. **Whether or not to Trial, which is related to the price of the Trial.**

   For a given price, should she Trial?

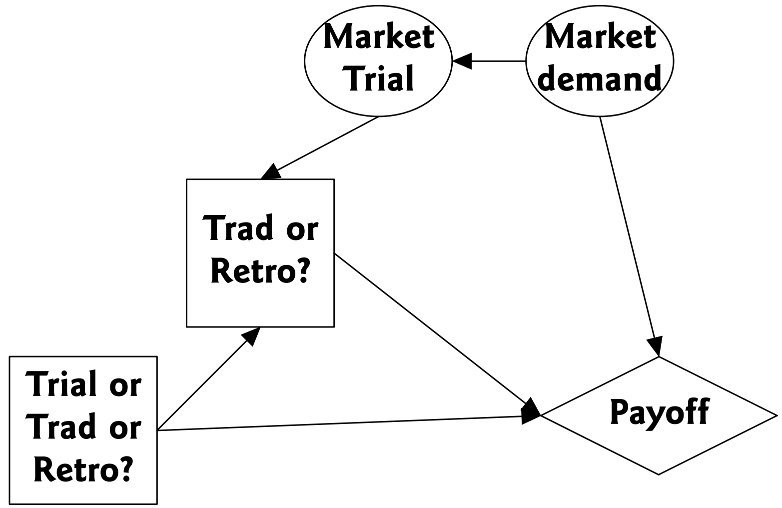
   If not, then the decision is as before: Trad or Retro?

2. **If she buys the Trial, what's the most she should pay for it?**

   To answer this, we need to examine her best choice with the Trial: Trad or Retro?

The *expected value of information* is the difference between Laura's expected returns *with* the Trial and *without* the Trial.

Laura's Influence Diagram: To Trial or Not?



The arrow from the Market Trial chance node to Laura's second decision represents the information (perfect or not) that she receives from the Trial.

That information in turn is influenced (perfectly or not) by the actual Market Demand.

If the Trial is 100% reliable, then there is no uncertainty after the Trial, and hence no arrow from the Market Demand uncertainty to the Payoff: all uncertainty is resolved before the Trad/Retro decision is made.

Laura: the VPI

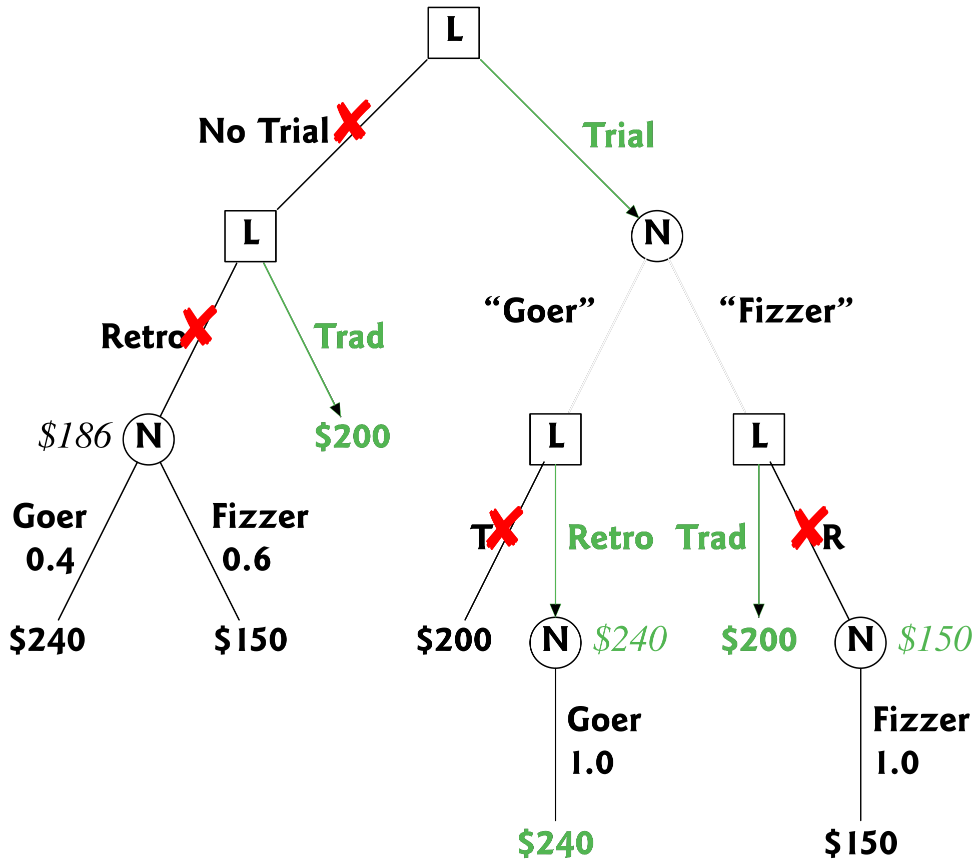
What Laura would like to know is what a specific piece of information implies for the eventual market demand for Retro, that is:

Probability (Retro is a Goer, given that Trial says “Goer”)

With perfect information, this probability is 1.

Q: What is Laura’s estimate of the probability that the Clairvoyant will say: “Goer”?

The new decision tree, including the test marketing decision:



The Shoe Decision with Perfect Information

(Remember: the Trial is 100% accurate.)

The new decision tree has four decision nodes □:

1. whether or not to test,
2. which range to choose without testing,
3. which range to choose if the test says "Goer," and
4. which range to choose if the test says "Fizzer."

The second, third, and fourth decision nodes are trivial:

2. Choose Trad if Laura chooses not to test. (Without testing, Trad pays $200k, which is better than the $186k expected from Retro.)
3. choose Retro if the test says it's a "Goer",
4. otherwise choose Trad,

How many chance nodes ○?

- Possibly three: the outcome of the test, and the 2 outcomes if she chooses Retro.
- But if the test is 100% reliable, it would rule out any uncertainty about Retro, one way or the other, and so the second and third chance nodes disappear.

***Question*: What is Laura's estimate of the probability of the 100%-reliable test coming up with Retro as a "Goer"?**

Well, her "prior" that Retro will be a Goer is probability = 0.4.
And *consistency* dictates that this is also her belief that testing will give the result that Retro is a "Goer".

- **Laura's expected return from the Trial:**

  \$240k × 0.4
  (the Trial indicated that Retro is a "Goer" and Laura chooses Retro)

  + \$200k × 0.6
  (the Trial indicated that Retro will "Fizz" and Laura chooses Trad)

  = \$216k

Laura's Expected VPI

- Her expected return of No Trial = \$200k from choosing Trad
  (which is higher than the expected return of \$186k of choosing Retro),

∴ The maximum Laura would be prepared to pay for the Trial is:

$$\$216k - \$200k = \$16k.$$

This is the *Expected Value of Perfect Information* in this decision;

The expected value of *im*perfect information would be less than \$16k.

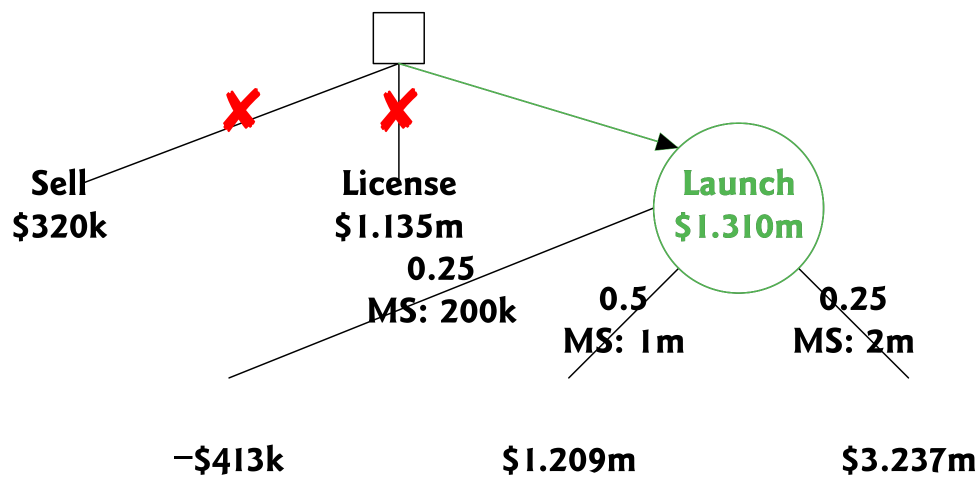
For an on-line applet for simple calculations of EVPI, see

http://www.cs.usask.ca/content/resources/tutorials/csconcepts/1999_6/Tutorial/Java/EVPIApp/evpi.html

1b. Calculating the VPI of the Glix case

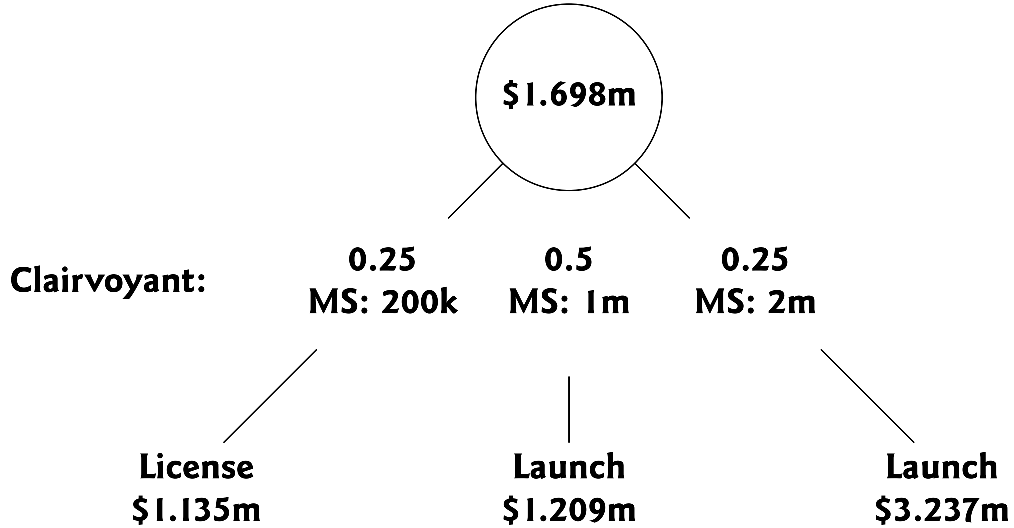
VPI is calculated by placing the uncertainty you want to evaluate before the decision. Then, recalculate the expected value.
Focus on Market Size (MS) uncertainty:
L = 200k, M = 1m, H = 2m.

Original tree: EMV = $1,310,910



Plot the Tree with Perfect Information

Tree with perfect information: EMV = $1,697,866



$\therefore$ **VPI = $1,697,866 − $1,310,910 = $386,956**

2. Probabilistic Sensitivity Analysis — Laura

Laura's belief in the probability p of Retro's being a Goer = 0.4

At what (cross-over) prior probability $\hat{p}$ would Laura choose Retro with No Trial?

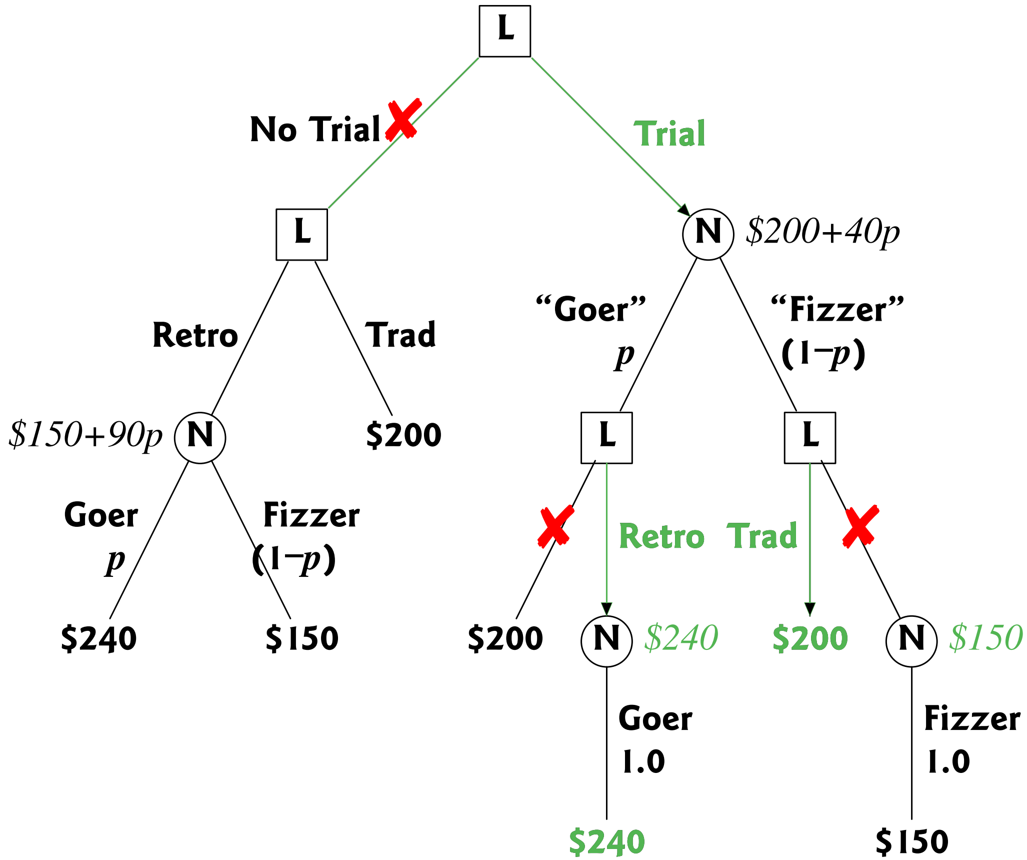
What would the expected value of a completely accurate Trial be then?

If Laura's probability that Retro is a Goer is p, then that must be her best guess as to the probability of the event that the Trial says Retro is a "Goer".

To be consistent, what else could she believe?

If she's uncertain about Retro's success, then she cannot be certain that a 100%-reliable Trial would say that Retro would, or would not, be a "success".

Laura's Decision Tree with Perfect Information:



The Shoe Decision with Perfect Information

(i.e. a 100%-reliable test)

The expected value of choosing Retro in the absence of a Trial:

$$\$150 + 90p,$$

compared with the unchanged value of \$200 of choosing Trad.

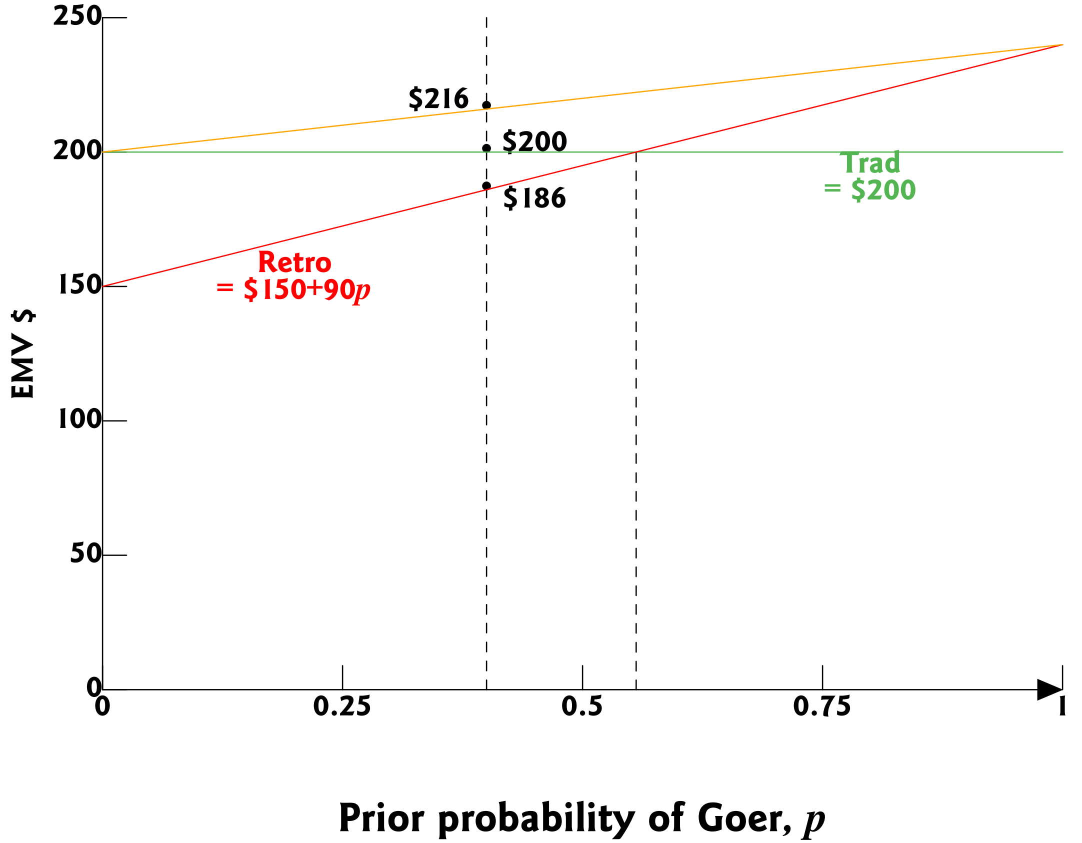
$\therefore$ the cross-over probability $\hat{p}$ is 0.556.

On the Trial side of the tree, to be consistent the probability of the test indicating that Retro will be a "Goer" must be p, and a "Fizzer" $1 - p$.

The expected value of choosing Trial =

$$200 + 40p.$$

Plotting these expected values as a function of the prior probability *p* of Goer:



Sensitivity Diagram: Expected Value against Prior Probability p of Goer

The Expected Value of Perfect Information (VPI) — Laura

The orange line: the expected value with the Trial.

∴ the minimum vertical distance down from the orange line to green or red line is the Expected Value of Perfect Information at any probability of Goer p.

At $p = 0.4$
- **the expected value of Retro is \$186,**
- **the value of Trad is \$200,**
- **and the value with Trial is \$216,**

The Expected VPI = the improvement in expected value with the Trial,

= the difference between the orange line and the next highest value, whether Trad or Retro.

= \$216k − \$200k = \$16,000 when $p = 0.4$

Certainty and the Expected VPI

When Laura is certain about the outcome with Retro, the value of reducing uncertainty is zero.

She is certain twice: when she

- **knows that Retro is a Goer (p = 1.0), or**
- **is certain that Retro is a Fizzer (p = 0.0).**

The cross-over probability $\hat{p}$ at which choosing Retro has a higher expected value than choosing Trad is 0.556.

Probability $\hat{p}$ corresponds to the highest expected VPI, and occurs when her decision is most sensitive to the probability p of Goer.

Highest expected VPI (@ $\hat{p}$ = 0.556) = $22,240.

Which variables are most crucial? — Laura

We have considered the decision's sensitivity to a *single variable*: the probability that Retro is a Goer.

But there might be some uncertainty about the payoffs of Trad and Retro under the two possibilities.

Which is the most critical variable on which to perform a sensitivity analysis?

Holding all other variables at their most likely values, one by one each variable be taken from its lowest likely value to its highest, and the effect of this on the optimand (the variable being maximised or minimised) be plotted.

A *Tornado plot*, with the variable with the greatest effect on top and that with the least on the bottom.

Those variables which can push the maximand lowest are the ones that should be subject to a sensitivity analysis.

3a. Laura's Case: The Expected Value of Imperfect Information

But what if the Trial is *not* 100%-reliable?

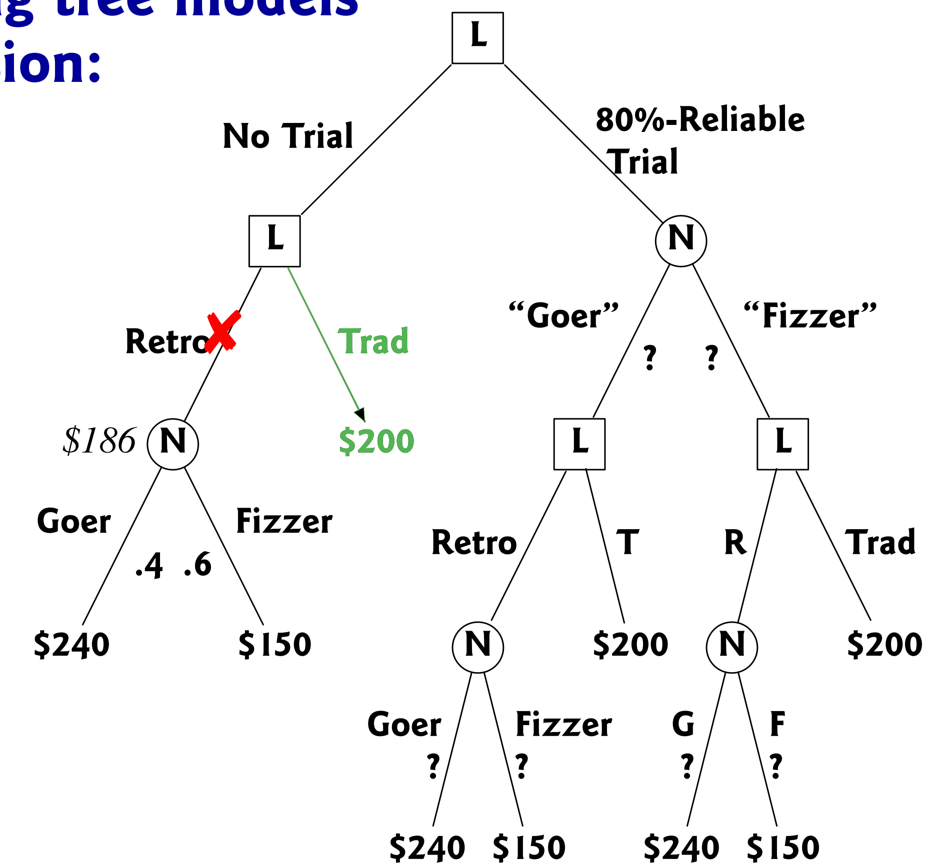
We'd like to know the maximum that risk-neutral Laura should pay for the test.

To answer this, we need to calculate two things:

- **Laura's probability that the unreliable test will indicate "Goer",**
- **and the *Conditional Probability* of Retro being a Goer if the test indicates "Goer".**

(With a 100%-reliable test, the former probability is 0.4 and the latter is 1.0.)

The following tree models Laura's decision:



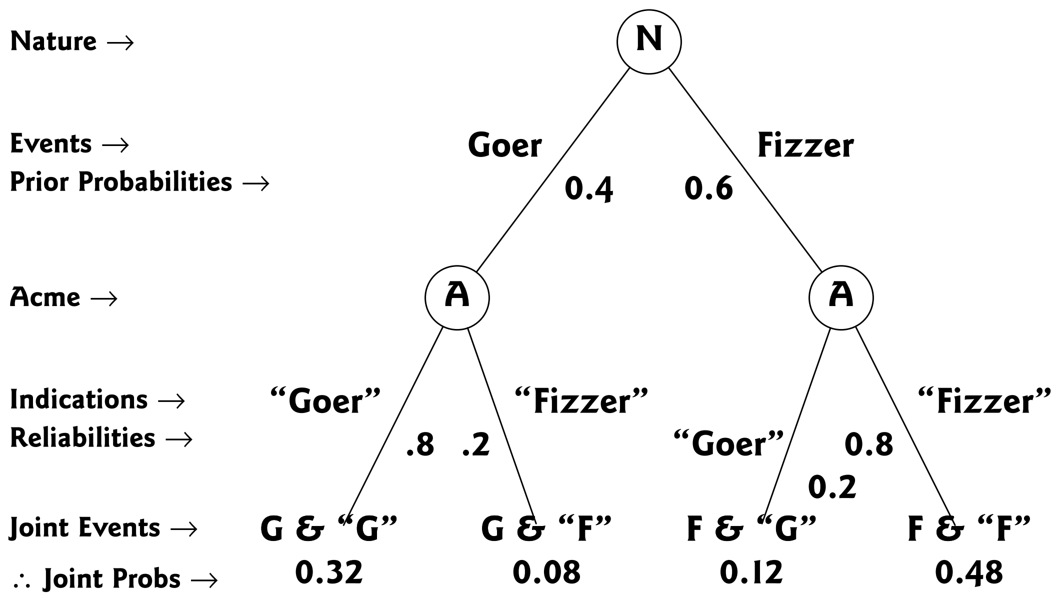
What probabilities do the question marks represent?

To answer this question we need to flip a smaller probability tree to calculate the *conditional probabilities*.

Laura and the Shoe Decision (cont.)

Laura decides to employ the Acme Marketing Company. Unfortunately, they are only 80% reliable:

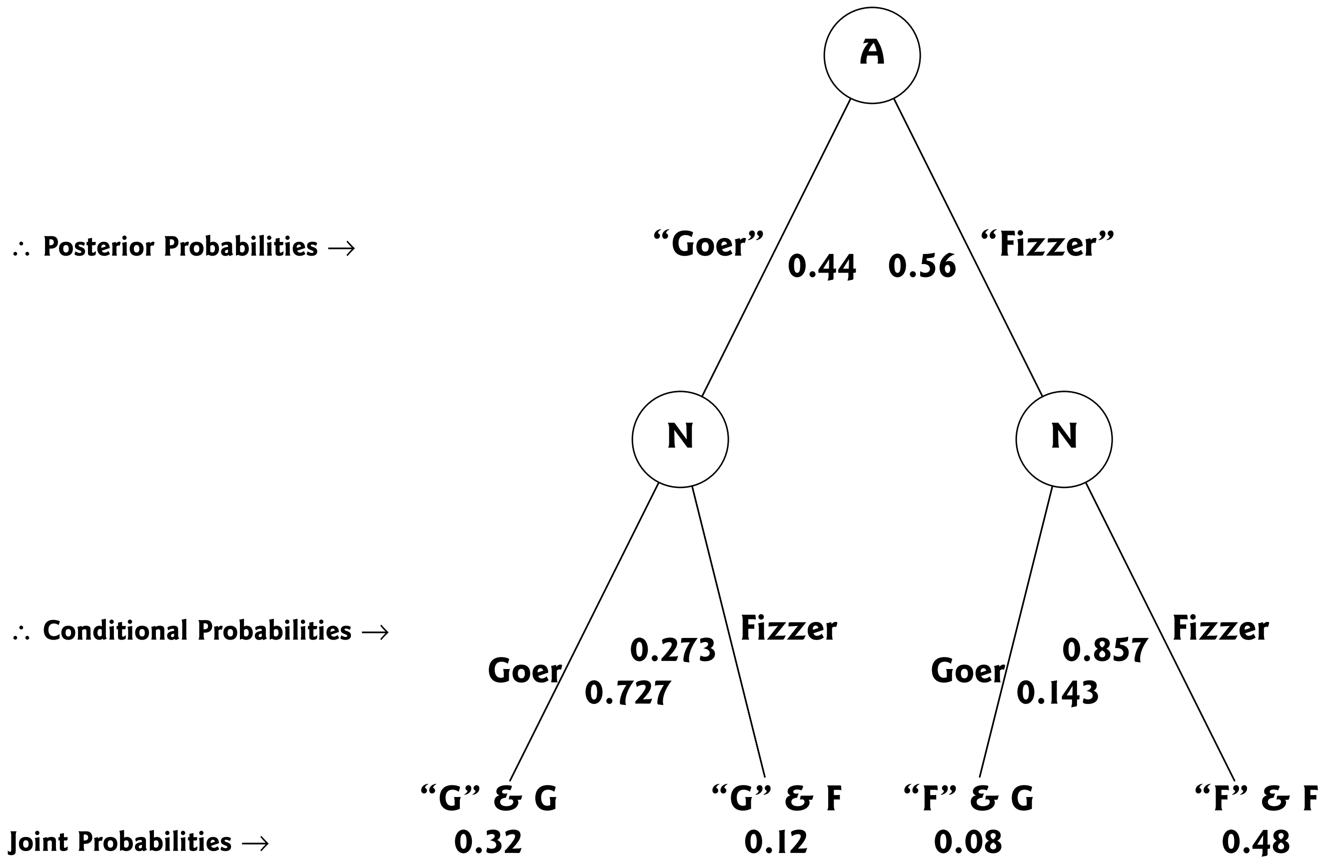
- **given that Retro is a Fizzer, Acme will say "Goer" 20% of the time (a false positive), and**
- **given that Retro is a Goer, Acme will say "Fizzer" 20% of the time (a false negative).**



Market Testing — a probability tree

The flipped probability tree: the Test result comes first.

"Flip" the above tree, to determine the chance of Retro being a Fizzer, given that the unreliable test indicates "Fizzer", and so on.



Market Testing

From the flipped probability tree:

- **the conditional probability of Retro being a Goer given that Acme says it's a "Fizzer" is** $\frac{0.08}{0.08+0.48} = \frac{1}{7}$ **= 0.143;**
- **the conditional probability of Retro being a Goer given that Acme says it's a "Goer" is** $\frac{0.32}{0.32+0.12} = \frac{8}{11}$ **= 0.727.**
- **based upon Laura's prior belief that Retro is a Goer with a probability of 40%, she expects that with probability 0.32 + 0.12 = 0.44 Acme will say "Goer".**

We can now replace the question marks in the decision tree above, which allows us to solve the decision problem, with expected values.
(Tree flipping gives the same results for *conditional probability* as using Bayes' Theorem.)

Expected Value of Imperfect Information — Laura

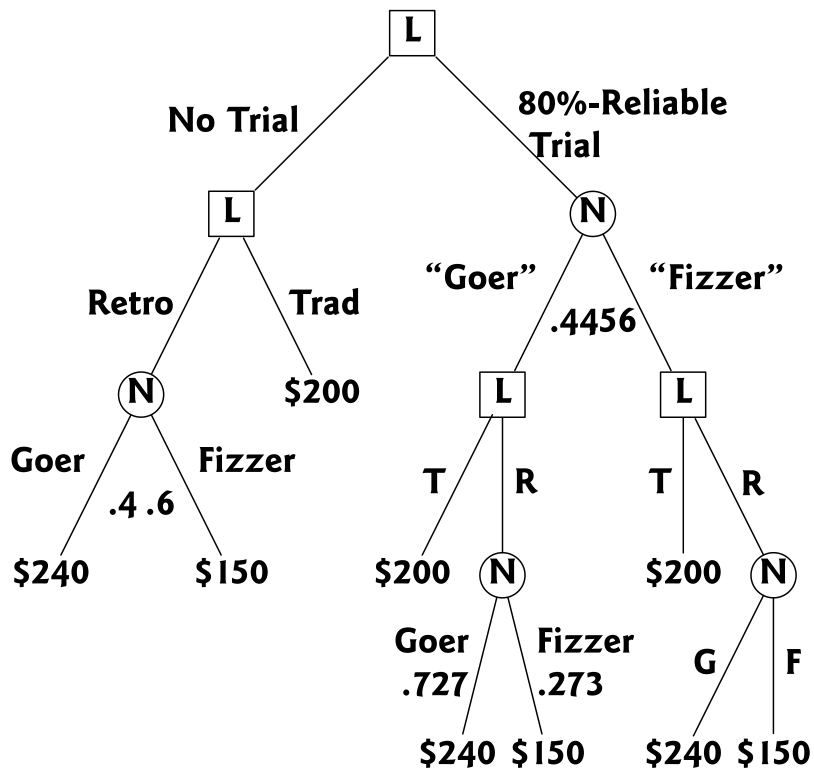
From the sensitivity graph above, $p = 0.44$ is less than the the cross-over probability $\hat{p}$:

- ∴ **If Acme says "Goer", which Laura expects will happen with probability of 0.44, then she will choose Retro.**
- & **Her expected payoff is $(150 + 90p) \times 1000$ = \$215,430, with the conditional probability that Retro is a Goer, given that Acme said "Goer", $p = \frac{8}{11} = 0.727$.**
- ∴ **If Acme says "Fizzer", which Laura expects will happen with probability of 0.56, then she will choose Trad, with a payoff of \$200k.**
- & **Her expected payoff with Acme's imperfect information is thus 0.56 × \$200k + 0.44 × \$215,430 = \$206,789.**
- ➢ **Her expected payoff without this information is \$200k, since she chooses Trad.**
- ∴ **The expected value to Laura of 80%-reliable information is \$206,789 − \$200,000 = \$6,789.**

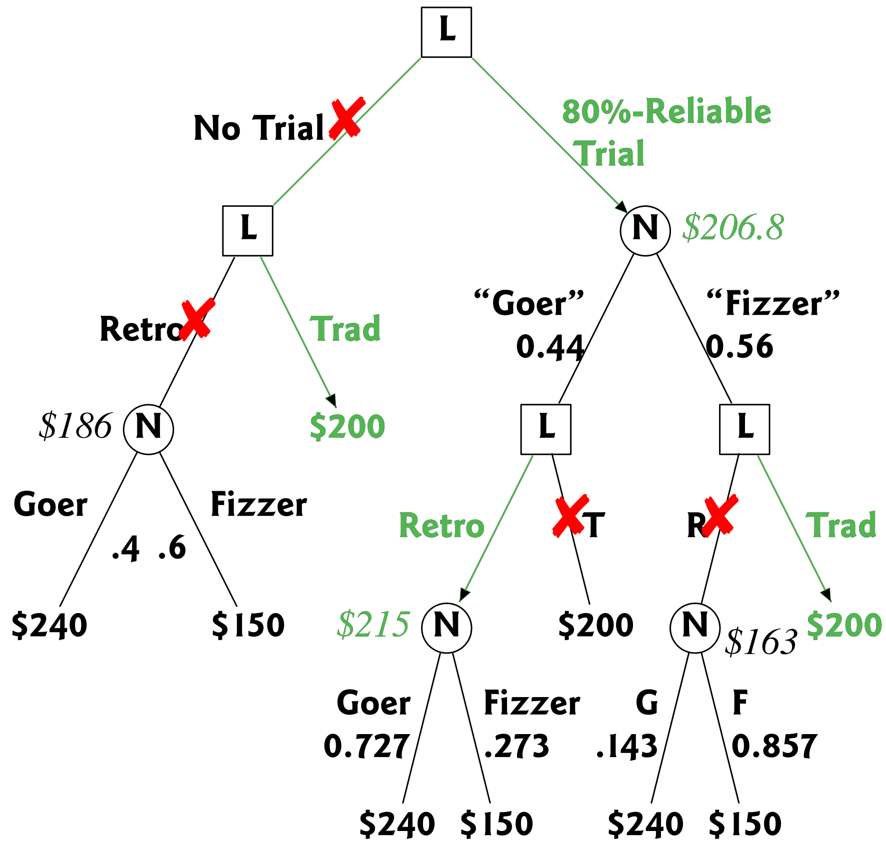
After tree-flipping:

- **Laura's *conditional probability* that Retro is a Goer, given that Acme has states that it will be a "Goer," is $\frac{8}{11}$ or 0.727.**
- **Laura's probability that Acme will state that Retro is a "Goer" is 0.44.**

Laura's full decision tree:



So the following tree models Laura's decision:



EV with the 80%-test = $206,790
EV without the test = $200k

∴ EV of the 80%-reliable information = $6,790

3b. Value of Imperfect Information of Glix:

We know the Value of Perfect Information is $386,956.

What if we could conduct a market survey for $300,000?

Would it be worth the investment?

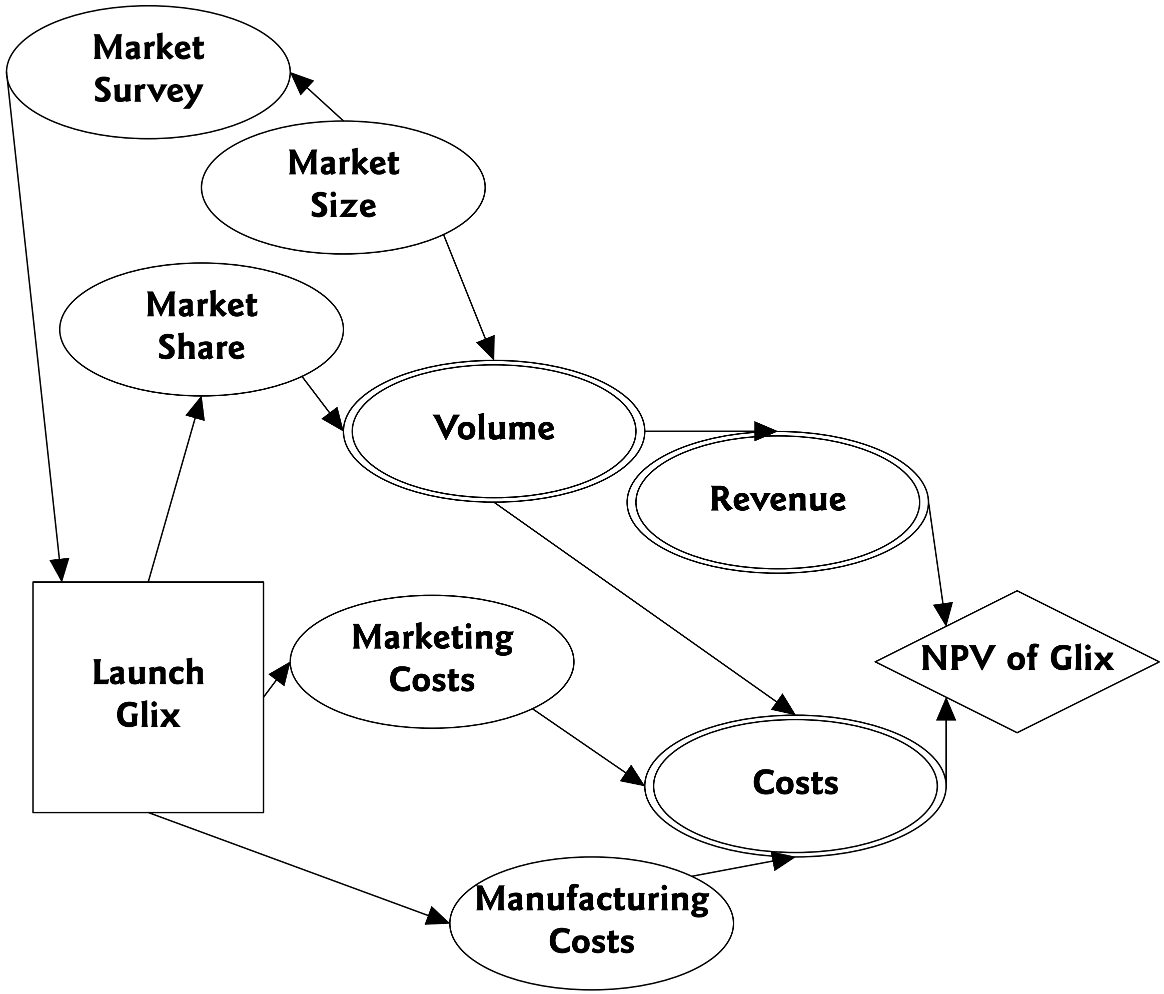
First, we must create a new influence diagram.

Notice that the Survey is influenced by Market Size rather than vice versa. This is to preserve the state of nature.

Recall: there are three possibilities for Market Size:

Low = 200,000
Medium = 1,000,000
High = 2,000,000

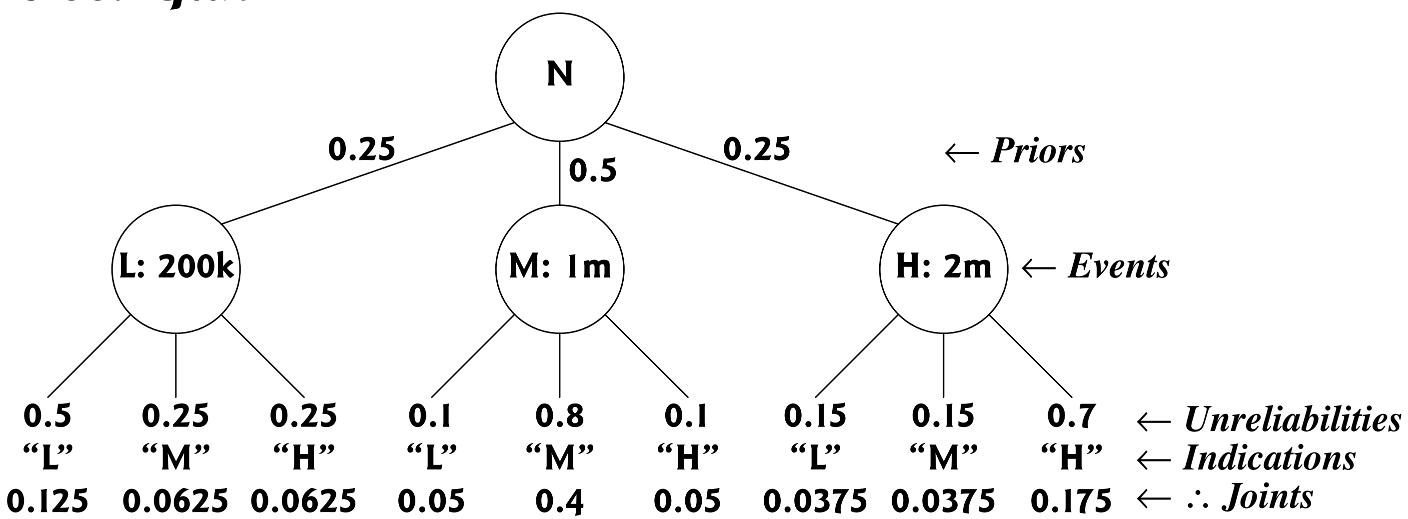
Glix's Launch? — Survey Influence Diagram.



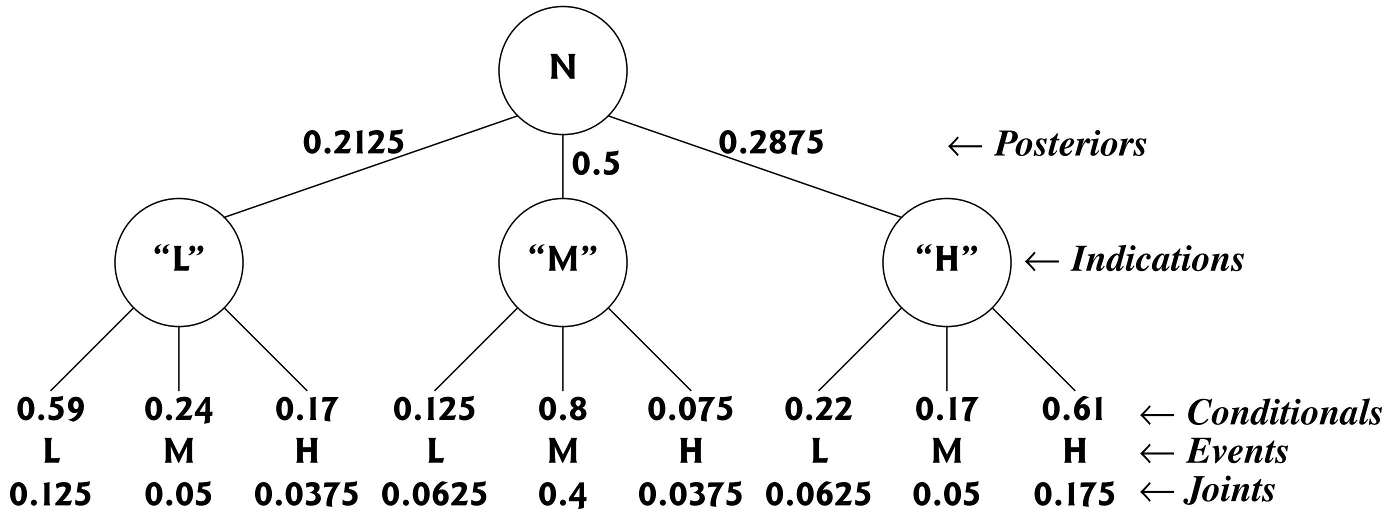
Glix: The Value of Imperfect Information — Reversing or Flipping the Tree

In order to calculate the value of imperfect information, we must flip the tree, to obtain the *conditional probabilities*, such as Prob (MS = 200k | the survey indicates "L"), which is correct.
***Assume*: if the survey is incorrect, then the two wrong indications are equally likely.**

Prior tree: Glix



The Posterior tree (flipped)



The posterior tree indicates that the Survey assessment is correct 70% of the time (the joint probabilities sum to 0.70): p(L&"L") + p(M&"M") + p(H&"H")= 0.125 + 0.4 + 0.175 = 0.70

We seek the Conditional Probabilities: given an Indication, how likely is the Event?

The flipped (posterior) tree also reveals:

1. **If the Survey indicates "L", then**
   $p_L = 0.59$, $p_M = 0.24$, $p_H = 0.17$,
   which means (from p. 9-13) the EMV(Launch | "L") = $596.8k
   → Licence = $1,135k should be chosen.

2. **If the Survey indicates "M", then**
   $p_L = 0.125$, $p_M = 0.8$, $p_H = 0.075$,
   which means the EMV(Launch | "M") = $1,158.4k > Licence,
   so choose Launch instead of Licence.

3. **If the Survey indicates "H", then**
   $p_L = 0.22$, $p_M = 0.17$, $p_H = 0.61$,
   which means the EMV(Launch | "H") = $2,089k > Licence,
   so choose Launch instead of Licence.

∴ The Value of the Survey for Glix:

The unconditional EMV with the Survey

$$= 0.2125 \times \$1.135\text{m} + 0.5 \times \$1.1584\text{m} + 0.2875 \times \$2.089\text{m} = \$1.420\text{m}$$

∴ The value of the Survey = $1.420m − $1.310m = $110k, which is the maximum that should be paid for the Survey.

Summary of Sensitivity Analysis and Value of Information

Decision analysis provides tremendous insight into the value of all the different alternatives, and can help to create new alternatives.

Sensitivity analysis is important in identifying the factors which affect the decision: the Tornado diagram.

Sensitivity to probability can help identify the variance that would cause you to change your decision.

The value of gathering additional information can be calculated *before* gathering the information.

Remember to consider the feasibility and reliability of gathering additional information. Just because you can calculate the value *does not mean* that you can either find the information or obtain it.

(Reading: Clemen, Reading 18 in the Package)